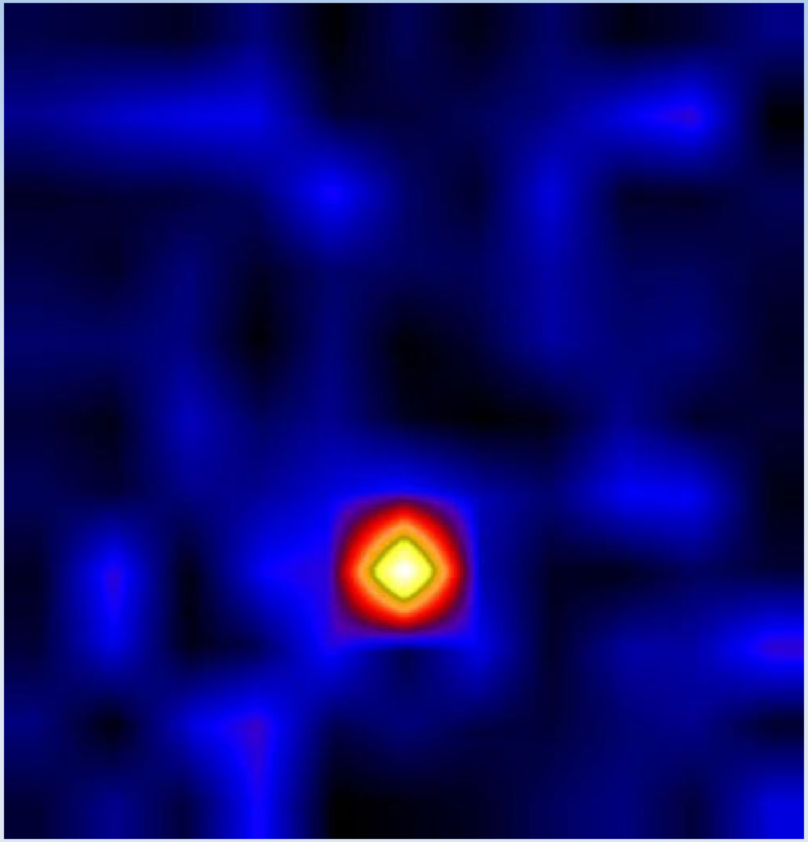
Compact Stars



Lecture 3

Summary of the previous lecture

- We have talked about close binary systems
  - Interacting binaries, their classification (detached, semi-detached or contact systems)
  - Definition of Roche lobe, position of Largrange points, fitting formulae for arbitrary mass ratio
  - The process of Roche lobe overflow, which leads to the formation of accretion disks
- I presented classification of binaries wrt. type of donor and accretor (compact star, evolved star, MS star)
- I will focus now on the basics of accretion disks in X-ray binaries. I will also present main evolutionary scenarios of XRBs (with black hole or neutron star as primary)

Circularization radius

- Circular orbit at R: Keplerian velocity

$$V_K = \sqrt{\frac{G M}{R}}$$

- Angular momentum is conserved

$$R_c V(R_c) = \frac{2\pi}{P_{orb}} R_{L1}^2$$

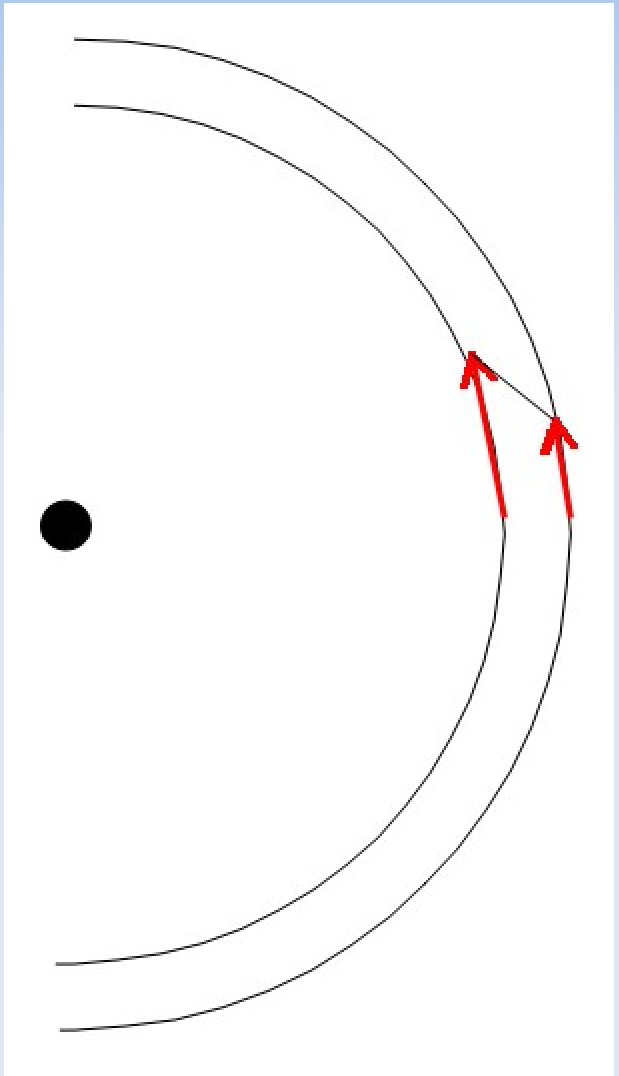
- Final formula $R_c/a = (1+q)(R_{L1}/a)^4$
- Approximately: $R_c/a = 0.0859\, q^{-0.426}$
- Various fitting fomulae, some giveR_{L2}: $q \rightarrow q^{-1}$

Formation of an accretion disk

- Ring of dR rotates differentially
- This leads to shear stress

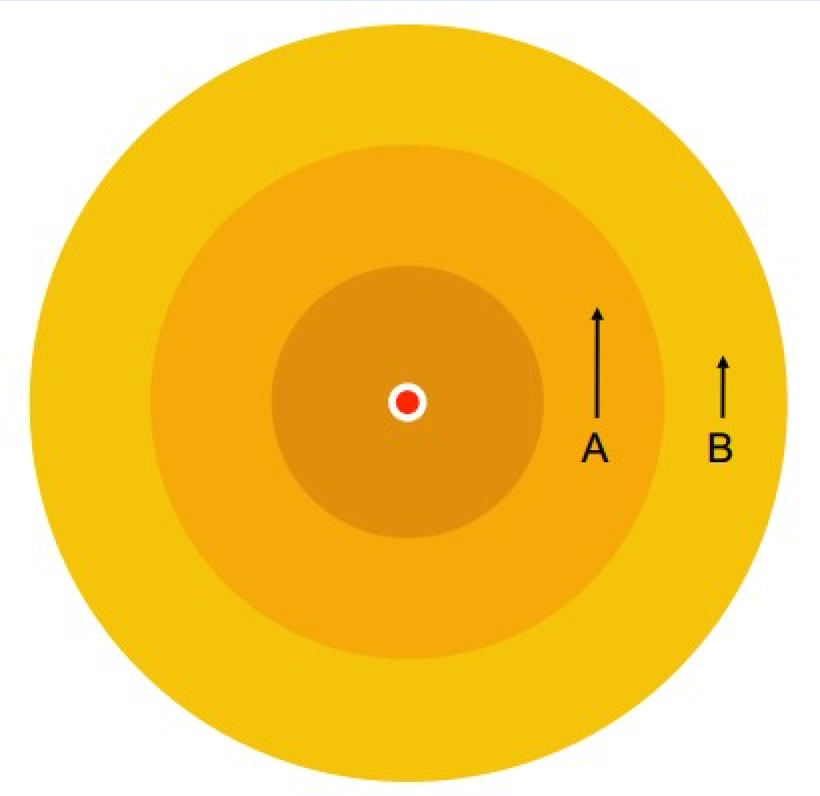
$$\Delta V = \frac{V}{2} \frac{\Delta R}{R}$$

- Friction opposes shear and causes the ring to spread inward and outward
- Angular momentum is transported outwards



Formation of accretion disk

- Subsequent rings are loosing angular momentum, but they are forced to stay on Keplerian orbits
- Rings are moving inwards
- Gravitational potential energy of the gas is liberated and disk heats
- Viscosity: responsible for angular momentum transport and disk heating



Viscous disk

- Viscous torque is exerted by the outer ring on the inner (and vice versa)

$$G(R) = 2\pi R \nu \Sigma R^2 \frac{d\Omega}{dR}$$

Where $\nu = \lambda v$ is the kinematic viscosity

- Dissipation rate

$$D(R) = \frac{G}{4\pi R} \frac{d\Omega}{d R} = \frac{9}{8} \nu \Sigma \frac{G M}{R^3}$$

Frank, King & Raine; Kato, Fukue & Mineshige textbooks.

Structure of the disk

- Disk is optically thick, geometrically thin.
- Height averaging → surface density
- Stationary: time derivatives vanish
- We solve the equations of radial mass conservation and momentum Euler equation.
- Adopt the inner boundary condition on the star's surface

Temperature as a function of radius

- The mass accretion rate

$$\dot{M} = 2\pi R \Sigma v_r$$

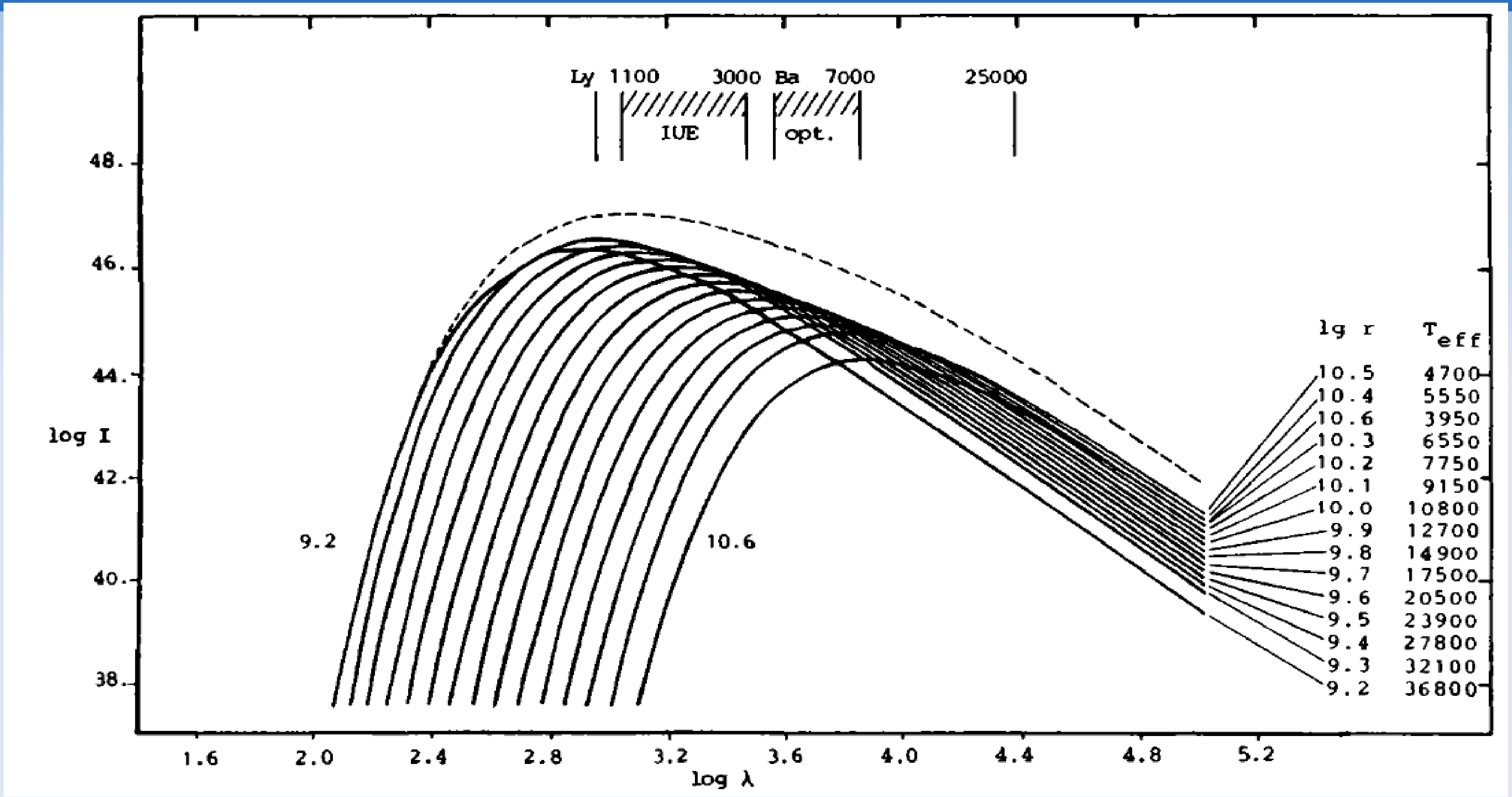
- From the angular momentum equation for steady disk we have

$$\nu \Sigma = \frac{\dot{M}}{3\pi}\left[1 - \sqrt{\frac{R_{star}}{R}}\right]$$

- Therefore the dissipation rate, so the locally emitted energy flux, is given by

$$F_{tot} = \frac{3 G M \dot{M}}{8\pi R^3} f(R) = \sigma T(R)^4$$

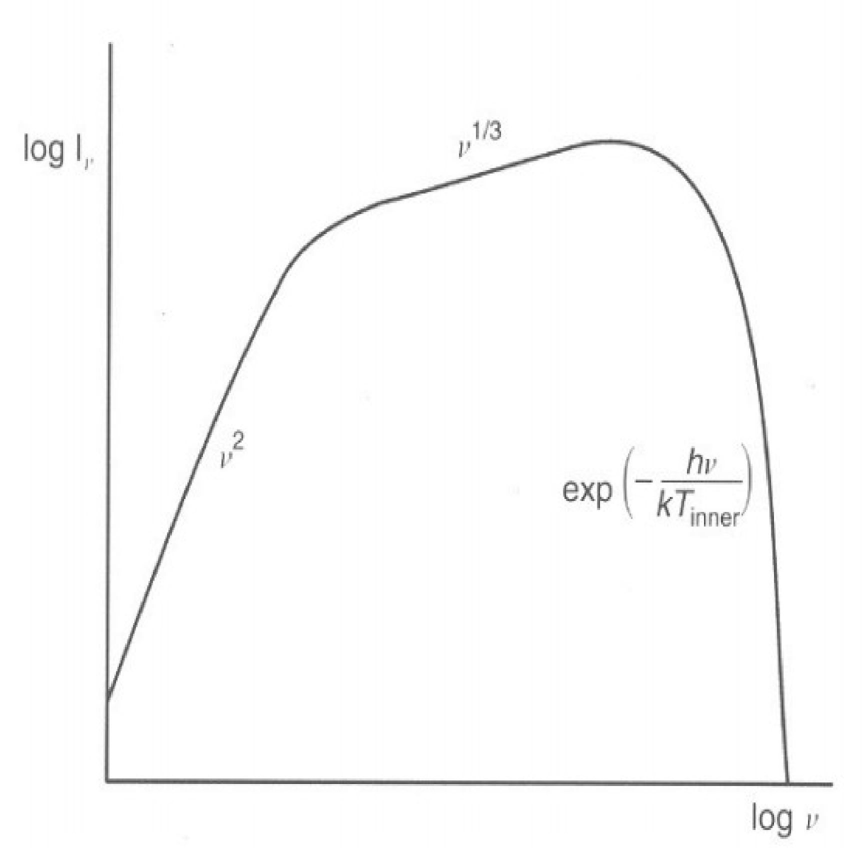
Disk black body



- Contributions from different annuli of the disk to its thermal spectrum

Shape of the spectrum

- From the outer parts, we'll see the Rayleigh-Jeans tail of the spectrum
- From the inner edge, we'll see the exponential cut-off



Disk energetics

- If the heat generated by viscosity is radiated, the luminosity of the disk is governed by the mass transfer rate and compactness of the accretor

$$L \propto \frac{G M \dot{M}}{R}$$

- Efficiency of this process is the fraction of the rest mass energy that is radiated

$$L = \eta \dot{M} c^2$$

Temperature and spectrum

- Average effective temperature: blackbody approximation

$$L_{disk} = \pi R^2 \sigma T_{eff}^4$$

- Thus the disk temperature scales with fourth root of mass and accretion rate, and inveresly with root of disk size
- The temperature profile must decrease with radius
- Larger disks are cooler

Examples

- Main sequence star: $\eta = 2 \times 10^{-6}$, (cf. The efficiency of H-He conversion is 0.007)
- White dwarf $\eta = 10^{-4}$
- Neutron star $\eta = 0.2$
- Schwarzschild black hole $\eta = 0.057$
- Extreme Kerr black hole $\eta = 0.4$

Break

Outer edge of the disk

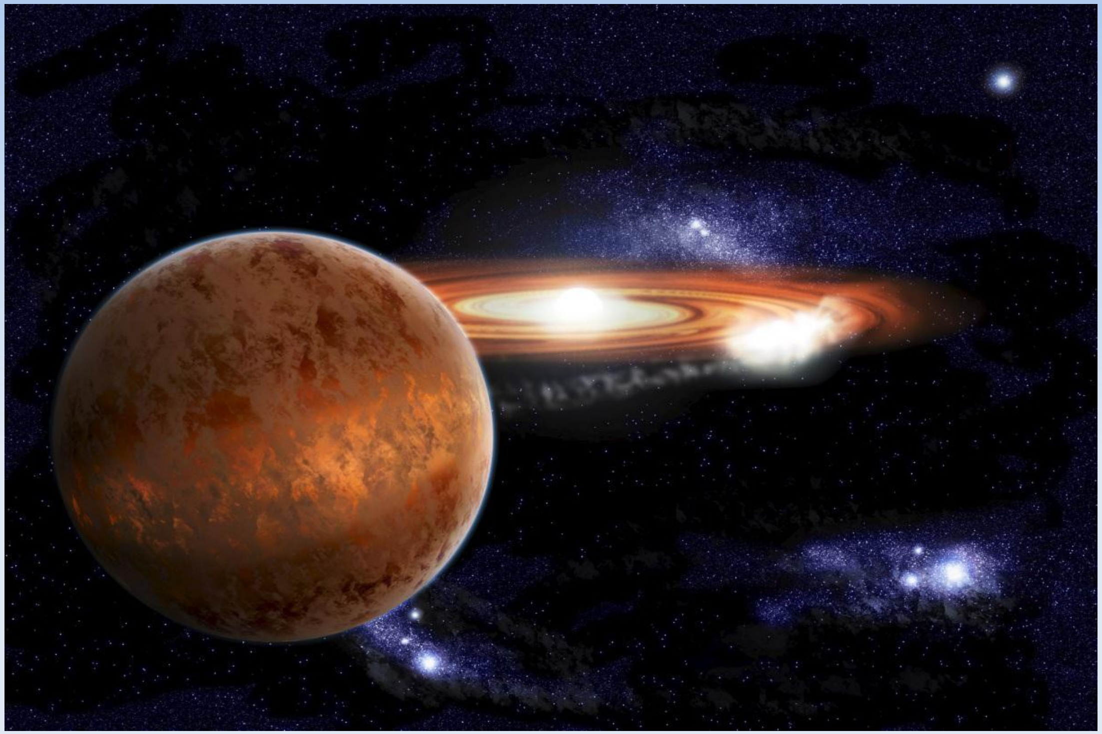
- Tidal interaction with the companion star keeps the disk from overflowing the Roche lobe.
- Paczyński (1977); Papaloizou & Pringle (1977)

$$\boxed{\frac{R_{max}}{a} = \frac{0.60}{1+q}}$$

- Combined effects of viscous diffusion, tidal dissipation and mass transfer stream: outer edge could reach up to 80-90% of the Roche lobe radius

Hot spot

- The stream of gas heats the outskirts of accretion disk with supersonic speed
- The shock-heated spot may radiate in Optics and emit more than the donor star and disk itself

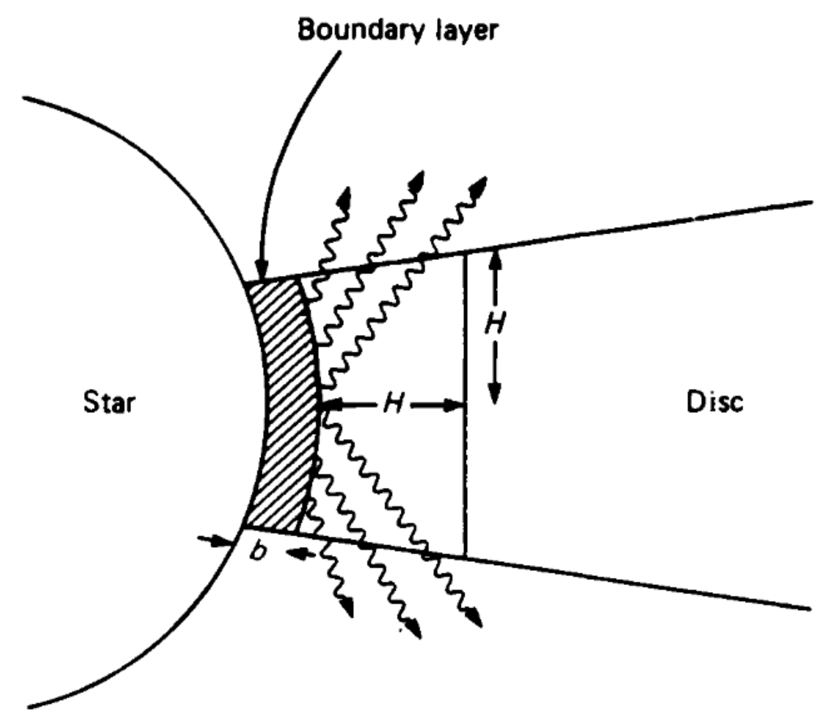


Inner edge of the disk

- Star's surface
- Black hole: radius of the marginally stable circular orbit, depends on the spin parameter
- Detailed discussion in Krolik & Hawley (2002) describes other 'working' definitions:
  - Radiation edge: innermost radius from which ignificant luminosity emerges; different from r_{ms} due to e.g. Gravitational redshift, photon trapping
  - Reflection edge: material even inside rms can reflect and reprocess X-rays
  - Stress edge: magnetic stress may continue well inside r_{ms}
  - Turbulence edge: MHD turbulence ceases

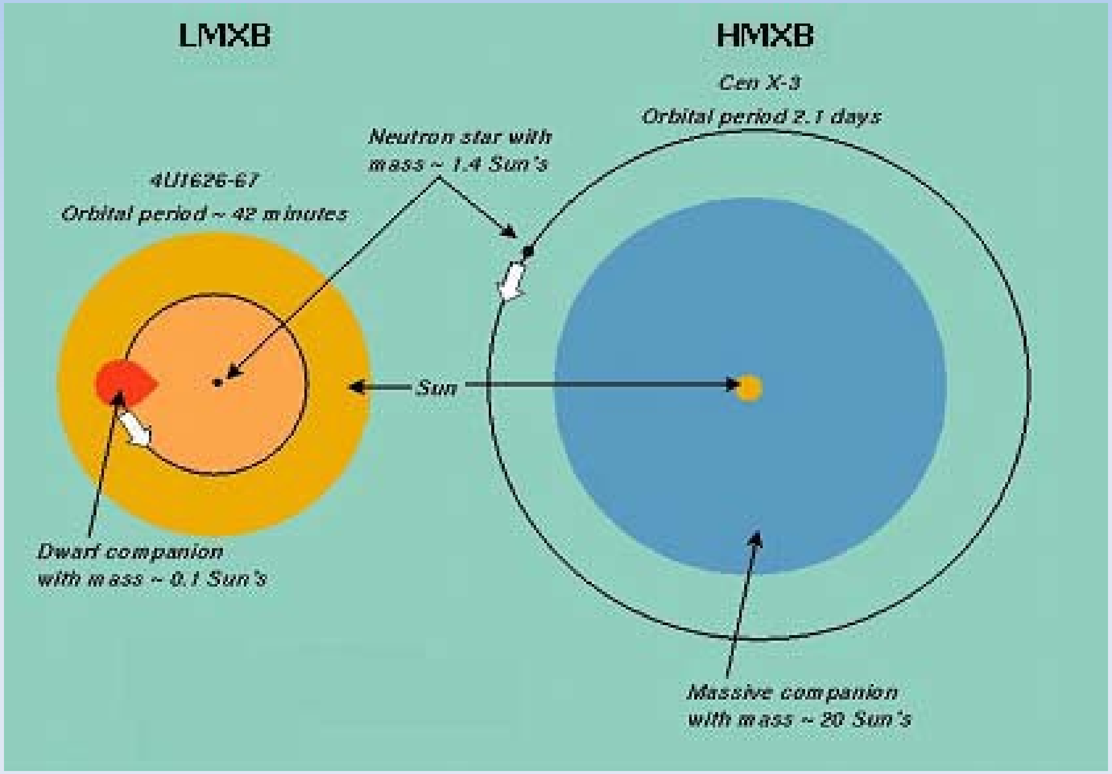
Boundary Layer

- Hard surface of the star (neutron star, white dwarf)
- Gas moving with Keplerian velocities in the disk must be decelerated to match the star's rotation
- Energy is used to spin up the star but is also dissipated
- Bulk of the boundary layer radiation is emitted in UV and X-rays

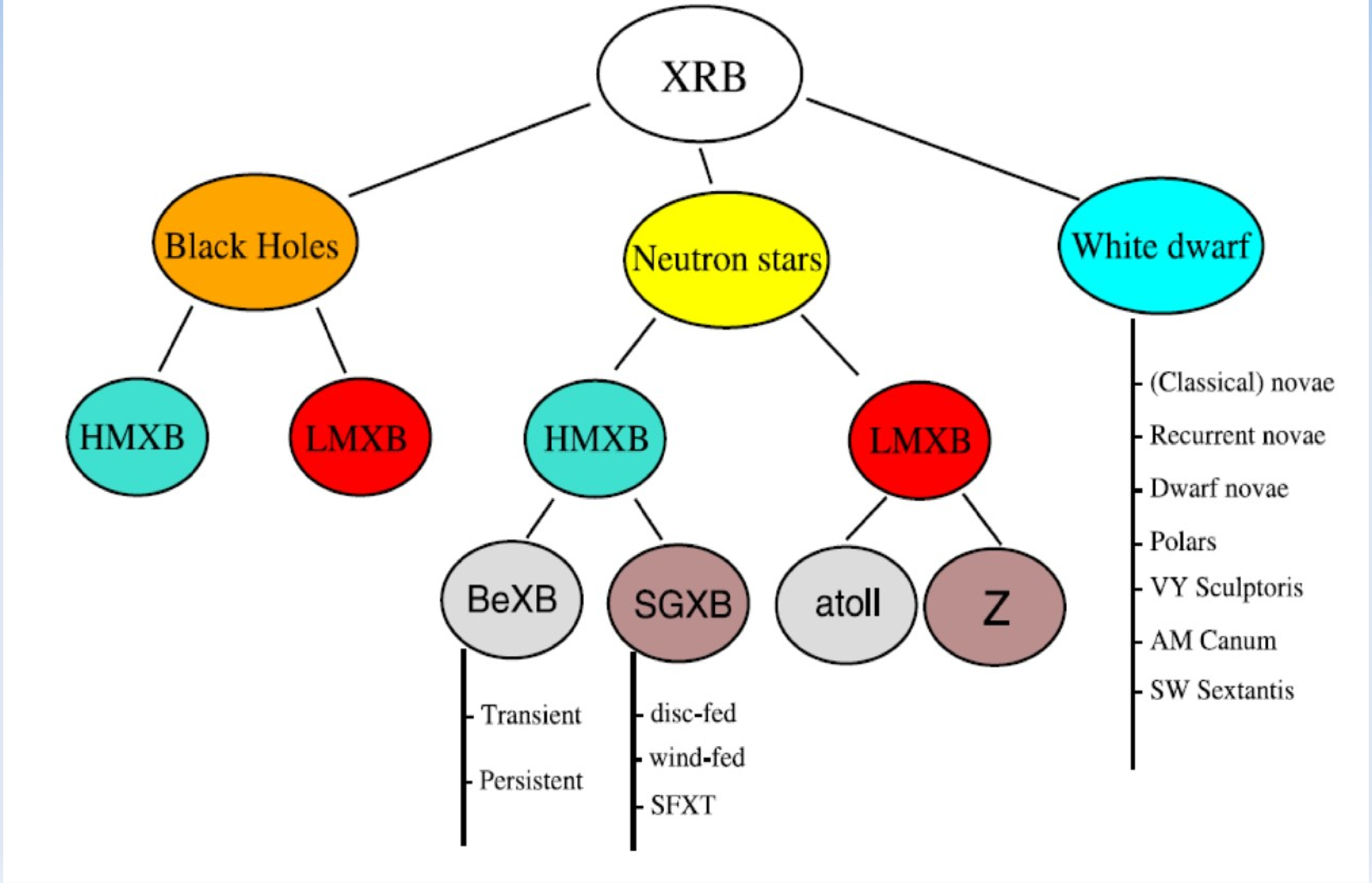


X-ray binaries

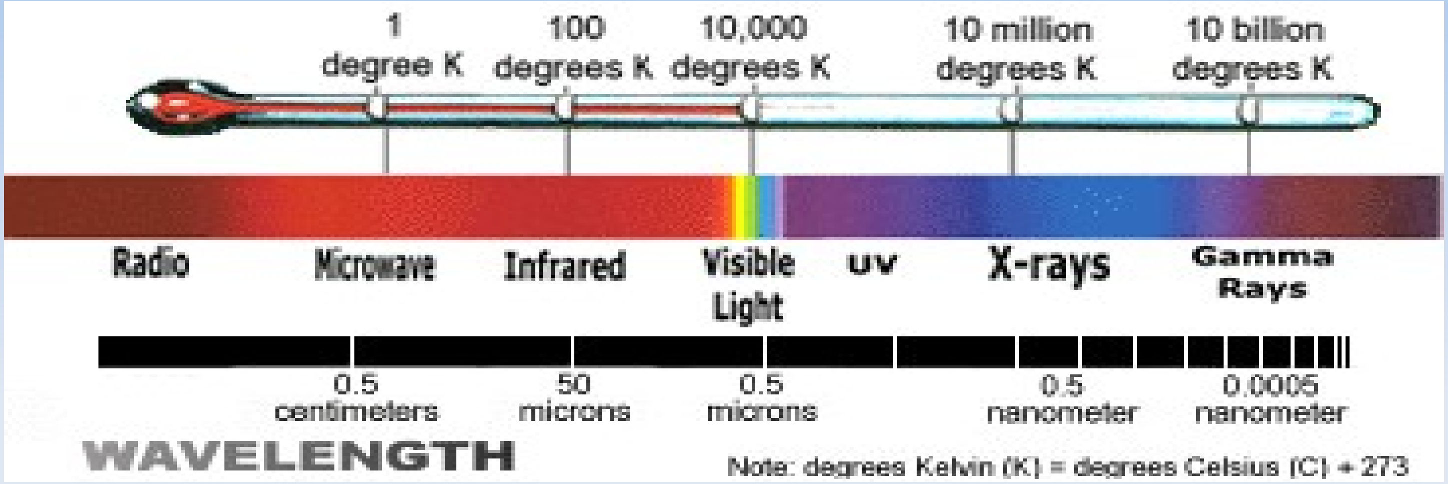
- Low mass
- High mass



Classification of X-ray binaries



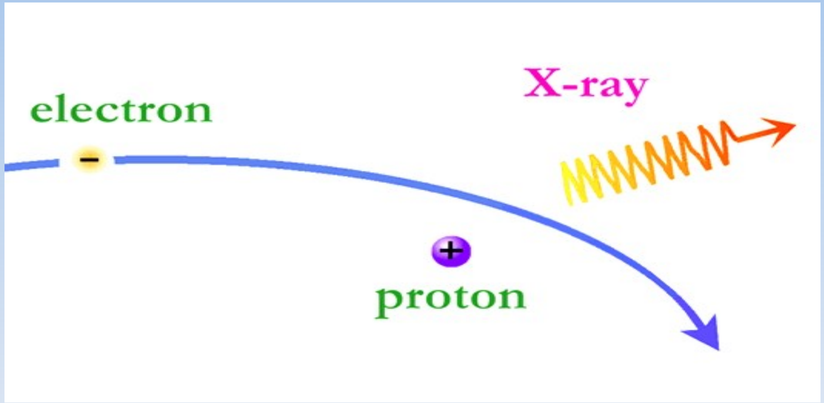
Radiation

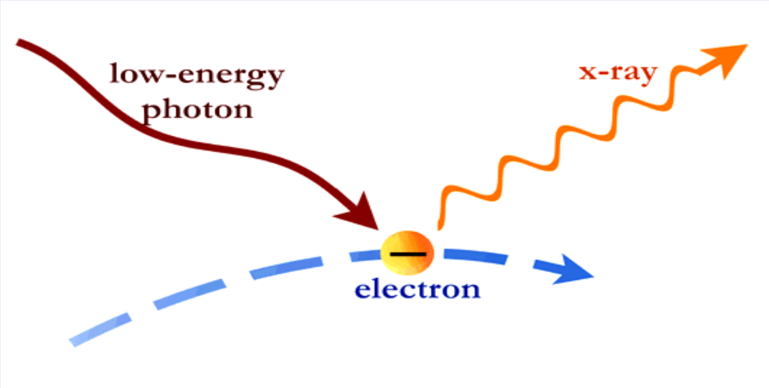


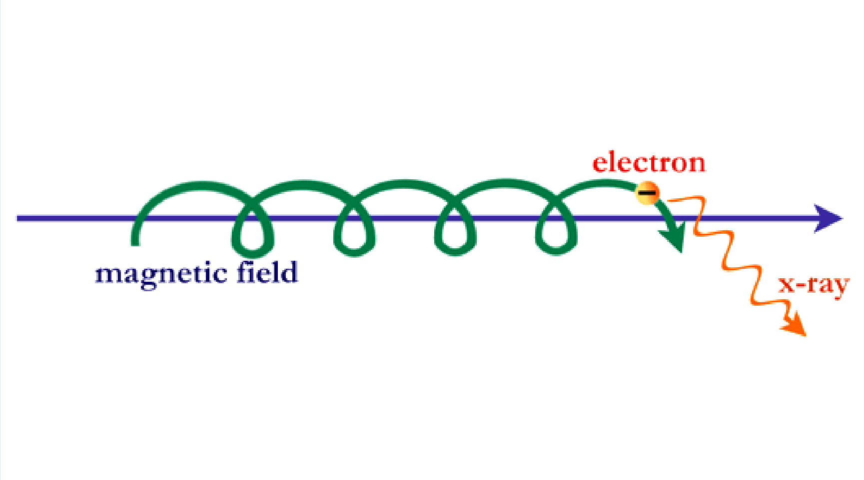
- Wavelength of X-rays: λ=1-25 Å, energy $h\nu$=0.5-15 keV

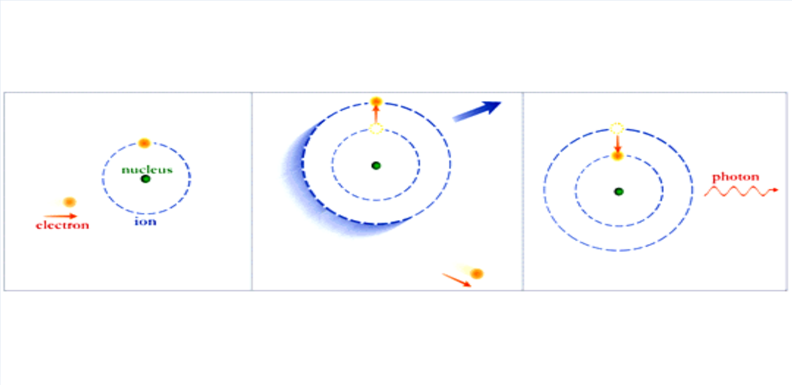
Mechanisms of X-ray emission

- Bremsstrahlung
- Compton upscattering
- Synchrotron
- Atomic emission









Next week

- More about observations of accretion disks in X-ray binaries
- Radiative processes, soft and hard spectra
- History of X-ray astronomy

- Suggested literature:
- Frank J. et al. ”Accretion Power in Astrophysics”,