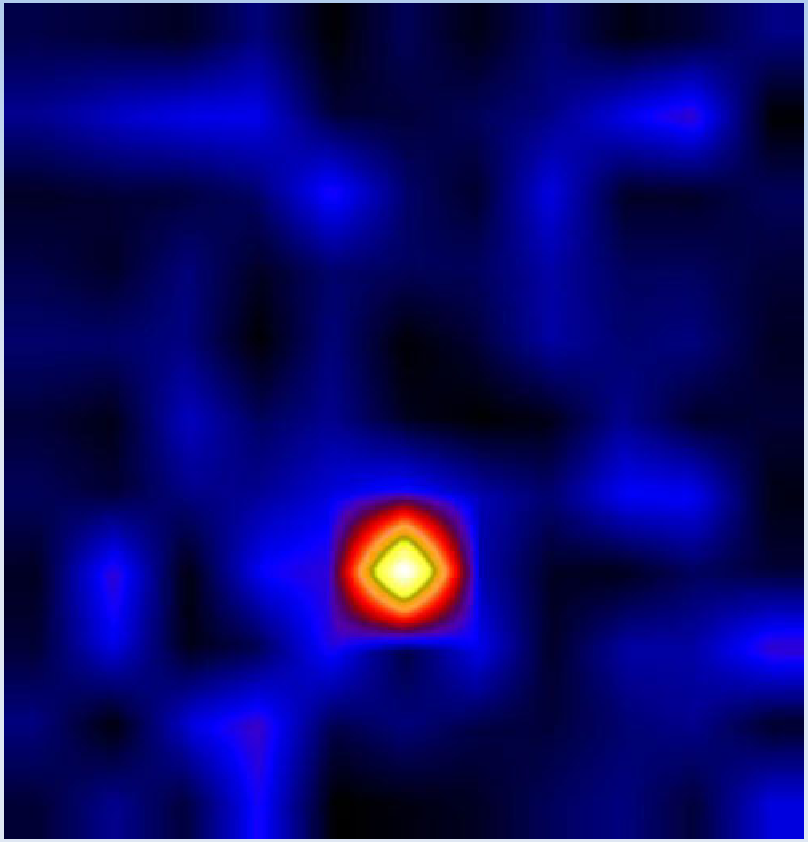
Compact Stars



Lecture 5

Summary of the previous lecture

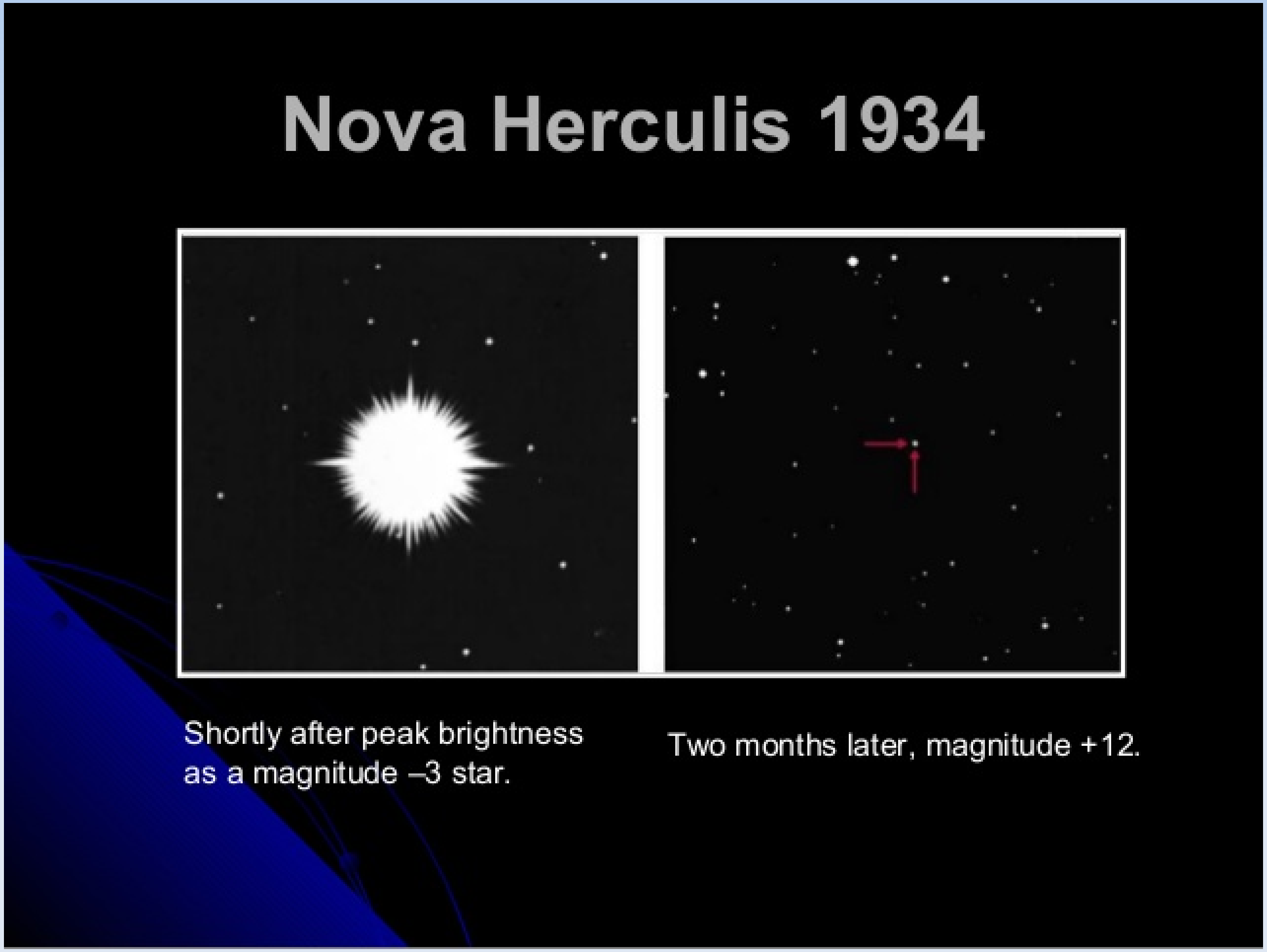
- We have talked about the observed properties of in X-ray binaries. The black body emission of accretion disk, the hard X-ray emission and radiative processes.
- I presented the milestones of X-ray astronomy history
- The classification of high and low mass X-ray binaries was supplemented with the evolutionary history of these systems

Today: state transitions, time-dependent accretion

- Today I will discuss the spectral diagnostics of X-ray binaries.
- I will show the formalism for steady-state and time-dependent accretion disk models. The radial and vertical structure equations.
- The thermal and viscous instability of accretion disk will be described, based on the alpha-disk theory.
- I will refer to the observations of Dwarf Nova outbursts as an example of this instability.

Novae

- Rate: 10/year
- Necked eye events: 3-5 per century
- Thermonuclear explosion on WD star



Nova Aquilae

It has remained constant ever since at about 10ᵐ8, approximately the same brightness it had before discovery. Visual estimates of the magnitude, made at frequent intervals since 1921 by Steavenson, and reported by him each year in the *Monthly Notices of the Royal Astronomical Society*, give no evidence of variability except for the gradual fading mentioned above.

Nova Aquilae (1918) is the only temporary star to date for which spectroscopic observations before the outburst have been made. In her discussion of the spectral development of the star, Miss Cannon[3] describes the prediscovery spectrum as follows:

"In order to determine, if possible, the spectrum of this object during the period, from May, 1888, to June 7, 1918, a careful examination was made of all the photographs of this region in the Harvard collection. Twenty photographs of spectra taken with the 8-inch telescopes were found which cover

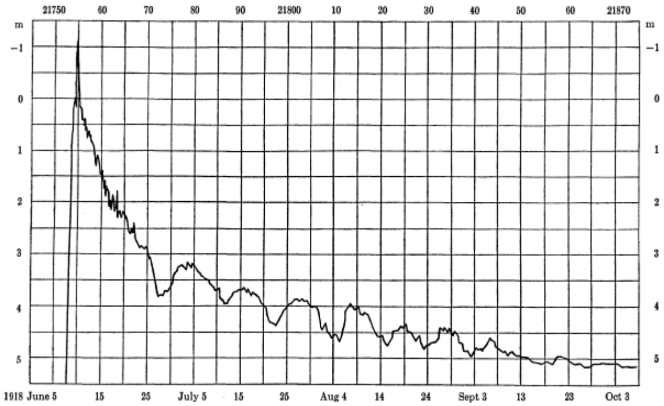
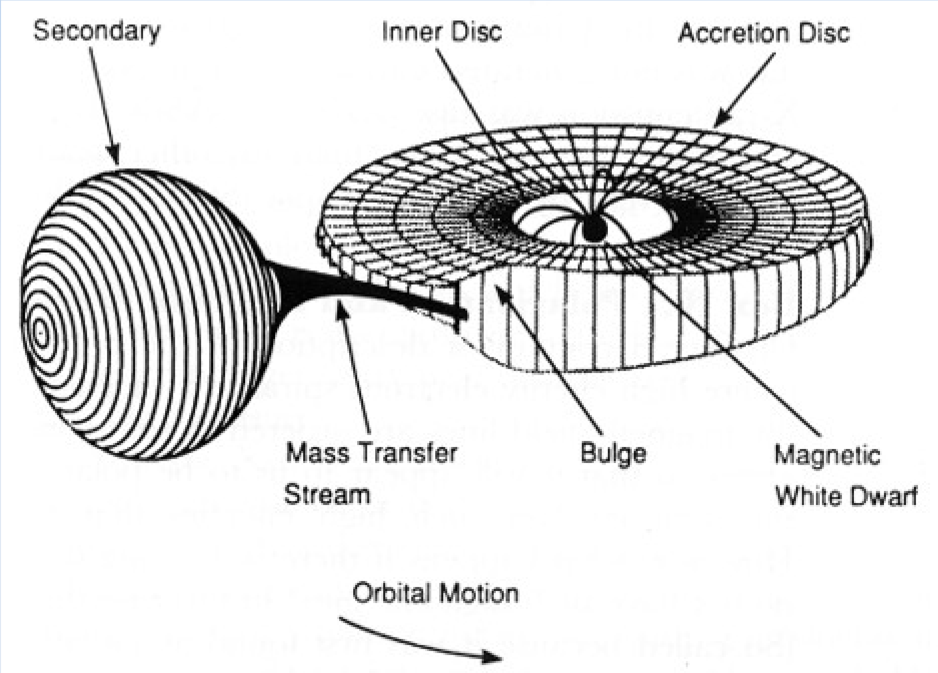


Fig. 1. Light-curve of Nova Aquilae (1918), by Leon Campbell. The Julian dates are 2400000 plus the numbers above the figure.

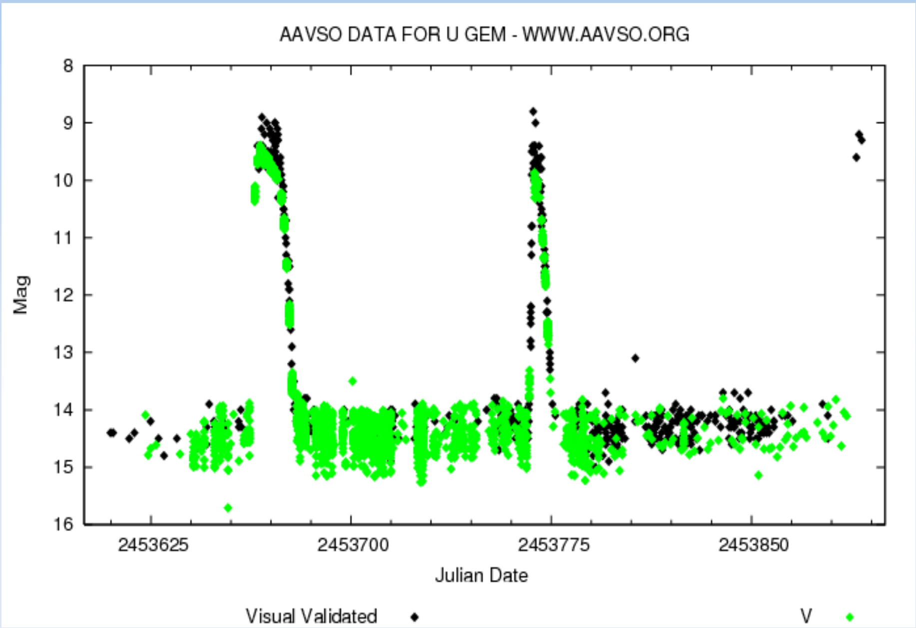
the region, and on about half of them, there is a faint image of the spectrum of this star. The best image is on the plate taken July 1, 1899. The spectrum appears to be nearly continuous, perhaps due to faintness. Several narrow dark lines are, however, barely seen, which appear to belong to the hydrogen series. In the distribution of light, the spectrum resembles those of Classes B and A. While the spectrum cannot be classified, it is safe to say that it was not of Class G or K, but was near Class A."

Mount Hamilton spectrograms.—When the word of discovery was received at the Lick Observatory, the spectrographs most commonly employed for the observation of novae were in Goldendale, Washington, where they had been taken for use in the Crocker Eclipse Expedition of June 8, 1918. Prior to their return on June 14, several spectrograms were obtained by various observers with the 3-prism Mills spectrograph in conjunction with the 36-inch refractor, and with the 2-prism quartz spectrograph attached to the Crossley reflector. Subsequent observations in 1918 were frequent; about half of these were made by Mr. G. F. Paddock, and the remainder by Messrs. J. H. Moore and H. Thiele. Most of the plates were taken with single-prism spectrographs having cameras of 16 inches focal length. One instrument covers the range from about 3850 to 5000A, and has a dispersion at Hγ of 58 A/mm; the other records from about 4800 to 6600 A, and has a dispersion at the D lines of 120 A/mm. The star was followed until 1918 October 25, when it was low in the western sky in the eve-

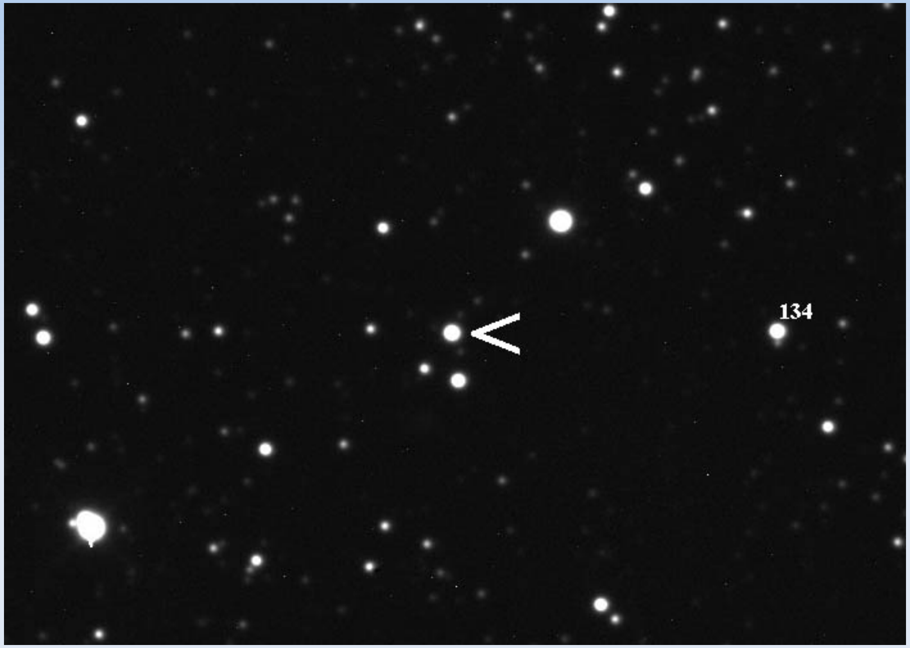
[3] *Harv. Ann.* **81**, 179, 1920.



Dwarf nova



U Gem dwarf nova (outbursts every 100 days)

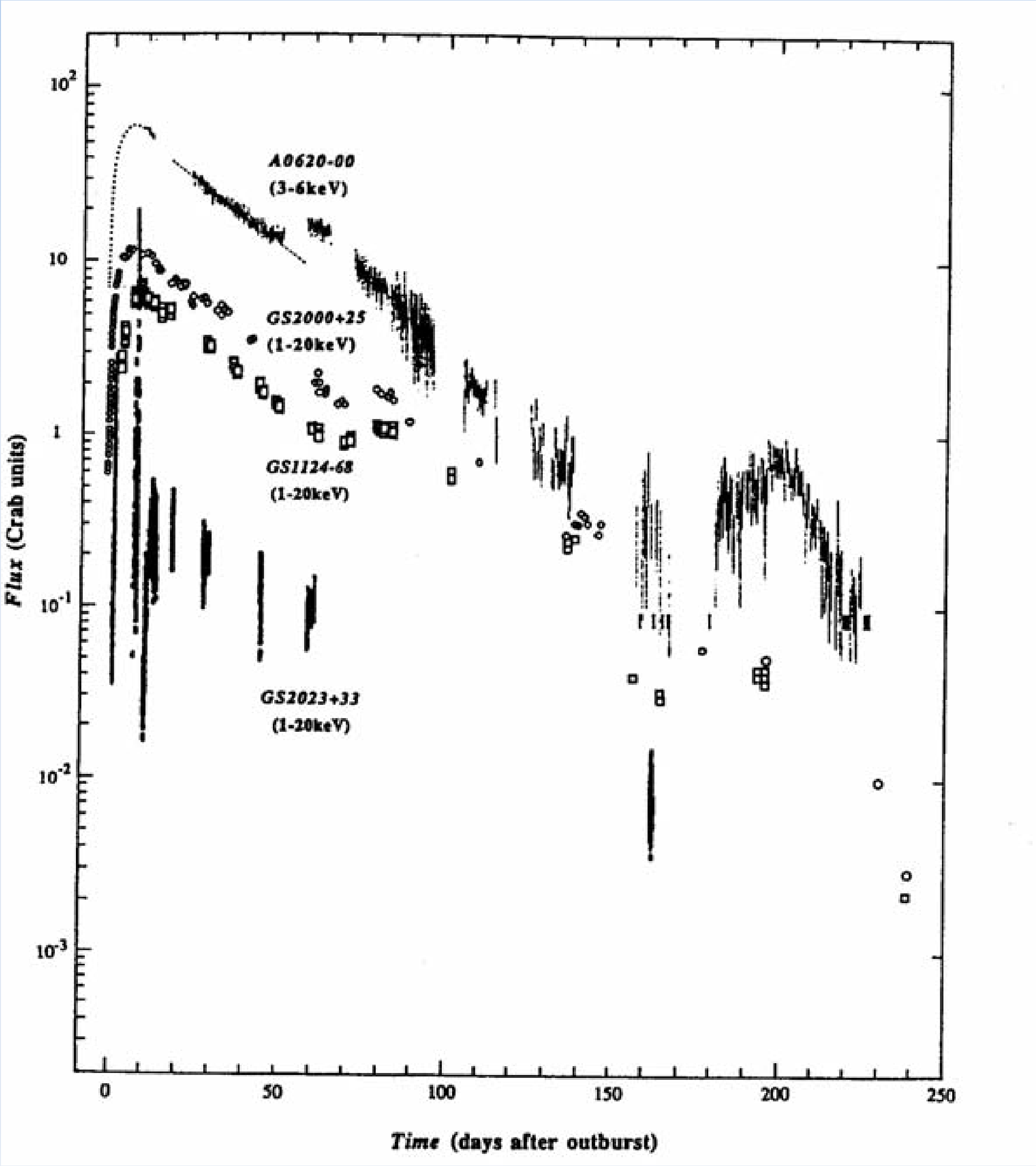


HT Cas dwarf nova (2010)

- U Gem, discovered in 1855, was thought to be a nova star.
- The mechanisms of outbursts for nova and dwarf nova are different!

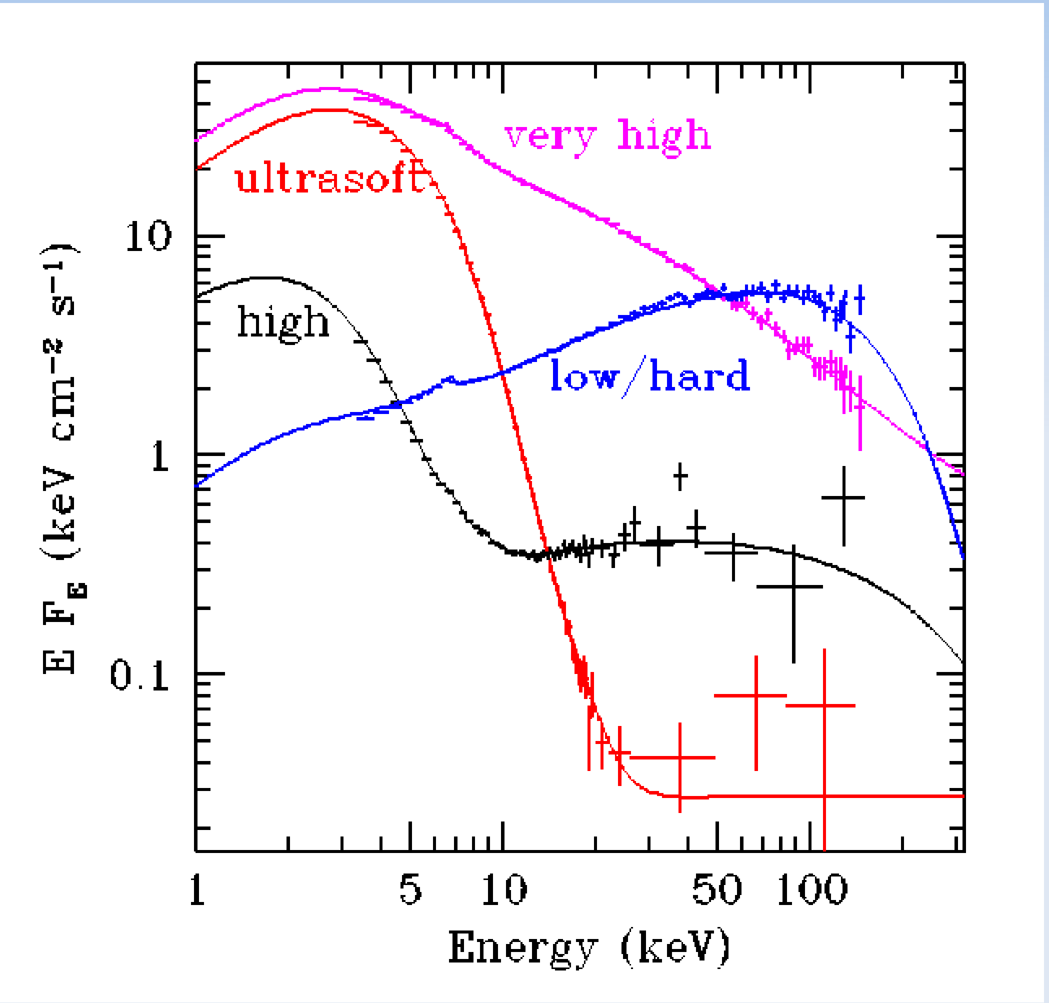
X-ray novae

- Flux increase by 2 orders of magnitude in several days
- Decline time of ~ months
- Many of them are recurrent
- Possibly outbursts are due to accretion disk instability

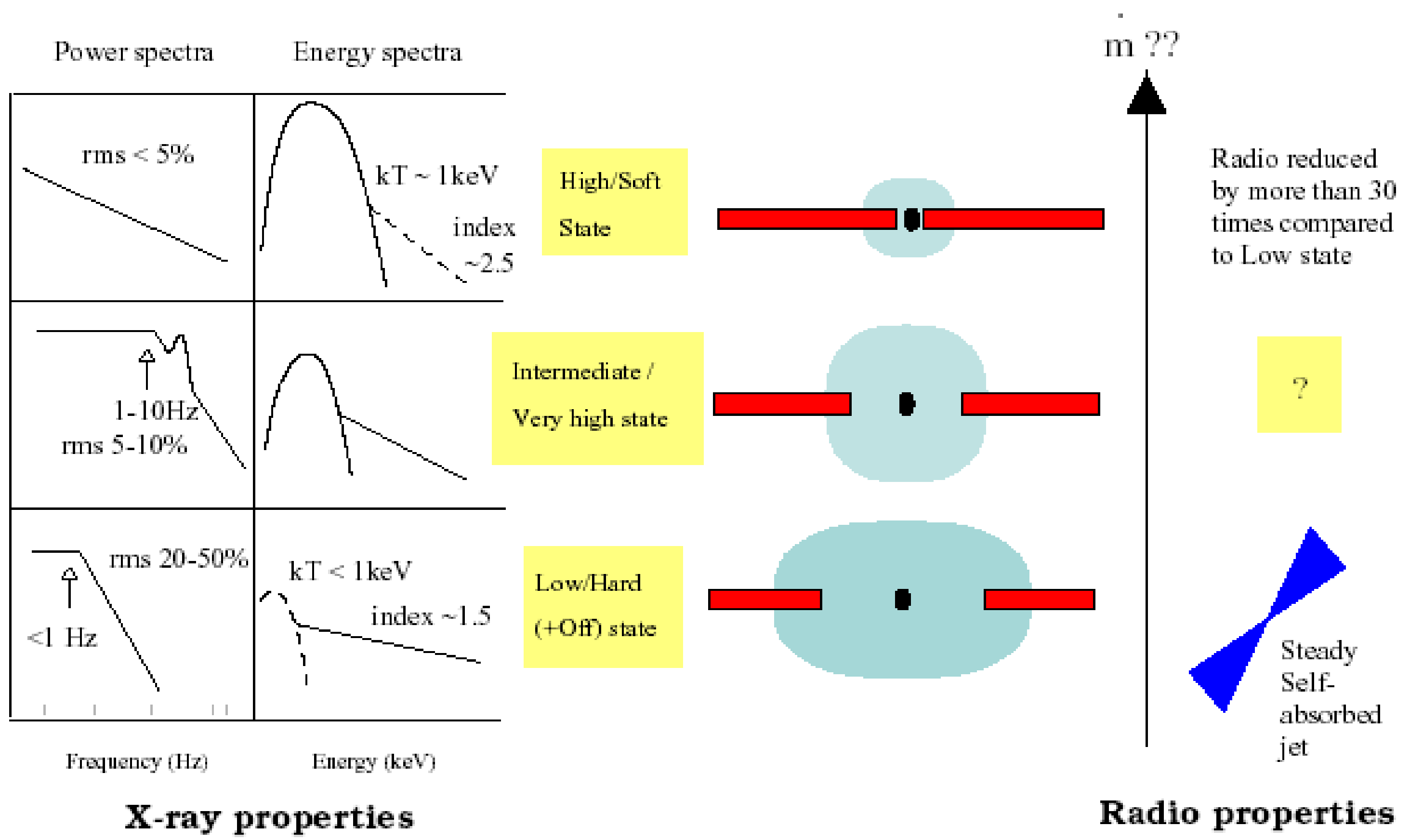


State transitions

- Spectra composed of disk black body component and power-law tail
- Hard X-rays possibly originate in a separate medium, i.e. Corona above the disk

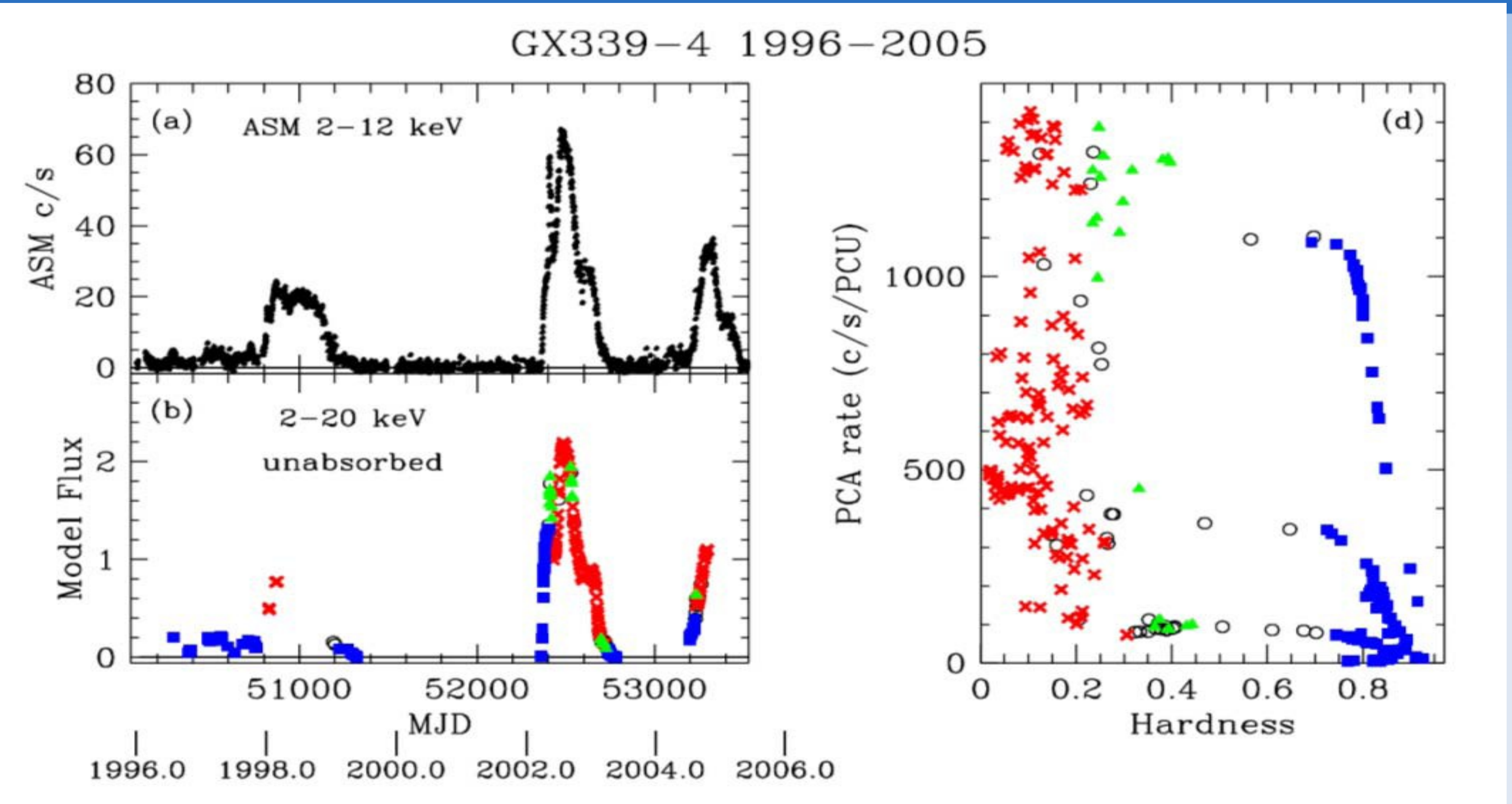


State transitions



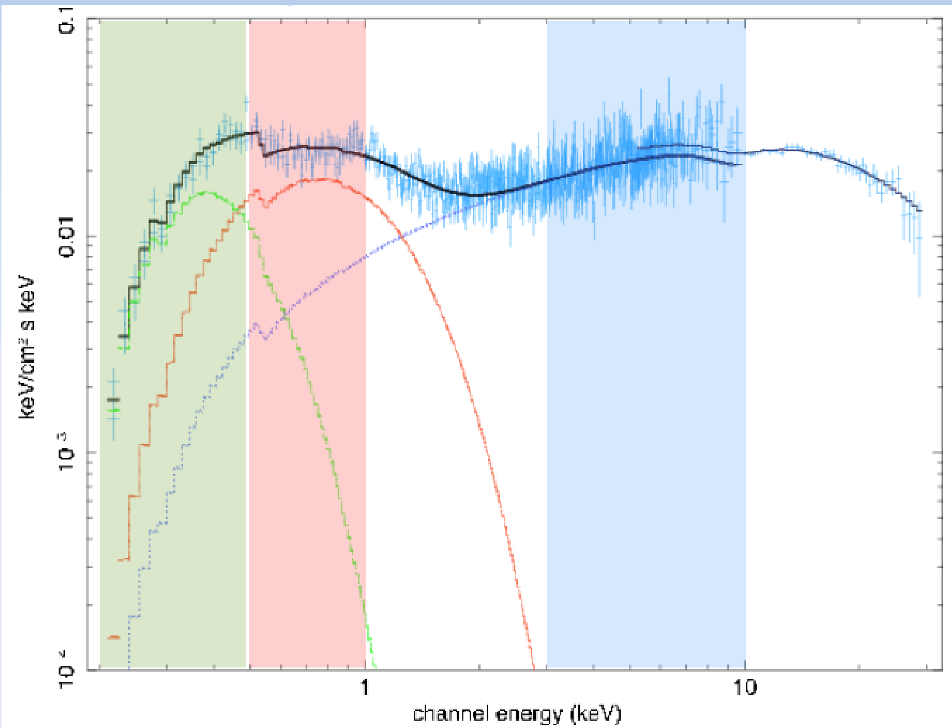
Fender 2000

Hysteresis effect

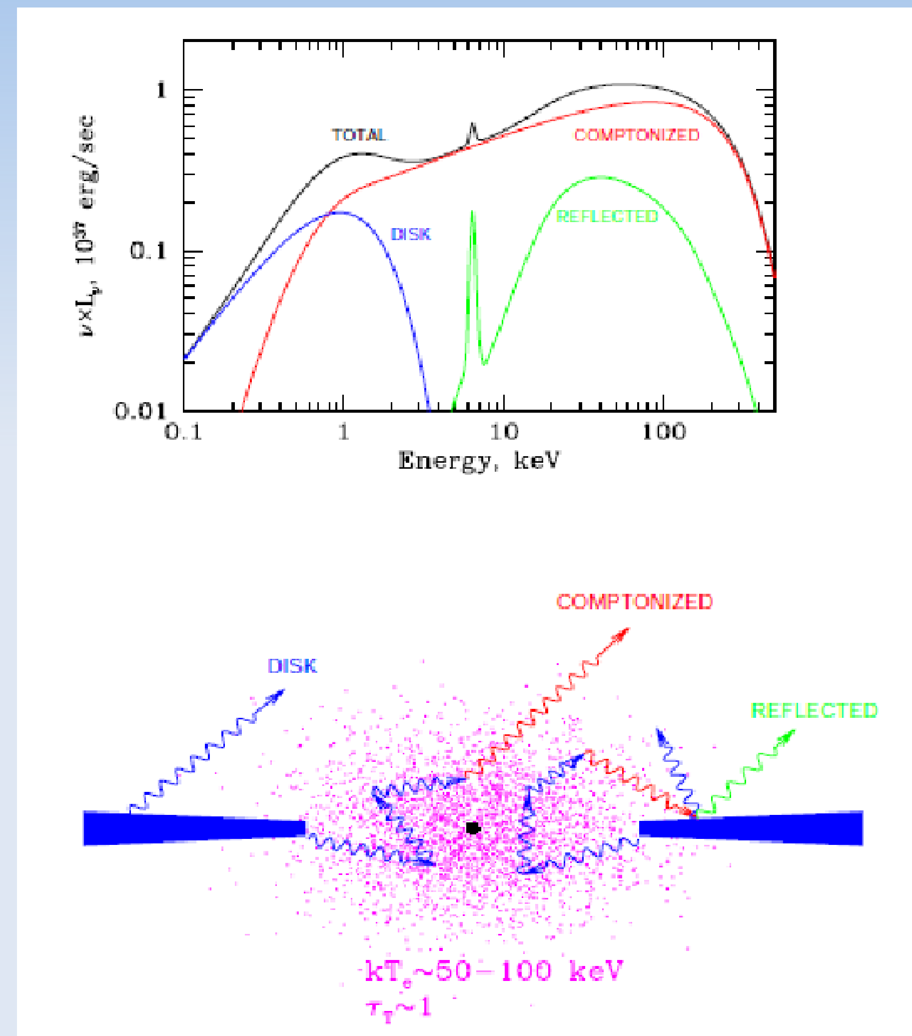


- Hard->soft transition is made at much higher luminosity than soft-> hard

X-ray spectral components



In August 2007, the X-ray binary pulsar XMMU J054134.7-682550 made a giant type II outburst, reaching the Eddington limit. The reflection of the hard X-ray, emitted close to the neutron star, on the inner part of the accretion disk allowed the determination of the geometry of the broadened inner disk (width of 75 km). The thermal emission from the disk could also be detected during the outburst.



Accreting black hole, Gilvfanov (2009)

BH diagnostics

- Lack of type 1 X-ray bursts
- In general, much softer spectra
- High energy power-law tail
- State transitions: high-soft and low-hard
- For a given orbital period, BH sources are ~100 times dimmer than NS sources

Comparison of X-ray characteristics

Feature	HMXBs	LMXBs
X-ray spectrum	Hard, kT >= 15 keV , or power-law index of 0-1	Soft, kT = 5-10 keV
Time variability	Regular pulsations, no X-ray bursts, often X-ray eclipses	Often X-ra bursts, quasi-periodic oscillations
Optical spectrum	Stellar-like	Reprocessing
Optical counterpart	Massive, early-type star (O, B), Lopt/Lx = 0.1-1000	Faint stars, Lopt/Lx = 0.001-0.01
Orbital period	1d - 1yr	10 min – 10 d
Distribution	Concentrated towards Galactic plane, age < 10^7 yrs	Concentrated towards Galactic center, age >10^9 yrs

Eddington limit

- Infalling matter is ionized hydrogen
- The upward force is exerted by the radiation flux, due to the Thomson scatterig on electrons

$$F_x = \frac{L_x \sigma_T}{4\pi r^2 c}$$

which is the number of collinsions per electron per unit time (σ_T = 0.66 x 10^{-24} cm^2), multiplied by photon momentum *p*

- These electrons communicate with protons by electrostatic coupling

Eddington limit

- Accretion occurs if the gravity exceeds the photon force.

$$F_{grav} = \frac{G M_X m_p}{r^2}$$

- The critical luminosity is therefore

$$L_{Edd} = \frac{4\pi G M_x m_p}{\sigma_T} = 1.3\times 10^{38} \left(\frac{M_X}{M_{Sun}}\right) erg\ s^{-1}$$

and is called Eddington limit (Eddington 1926)

- The same limit applies to massive stars supported in hydrostatic equilibrium by radiation pressure

Magnitude of viscosity

- Viscous stresses are generted via thermal and turbulent motions
- In cylindrical coordinates, the viscous stress tensor rϕ component is

$$T_{r\phi} = -\rho \nu r \Omega'$$

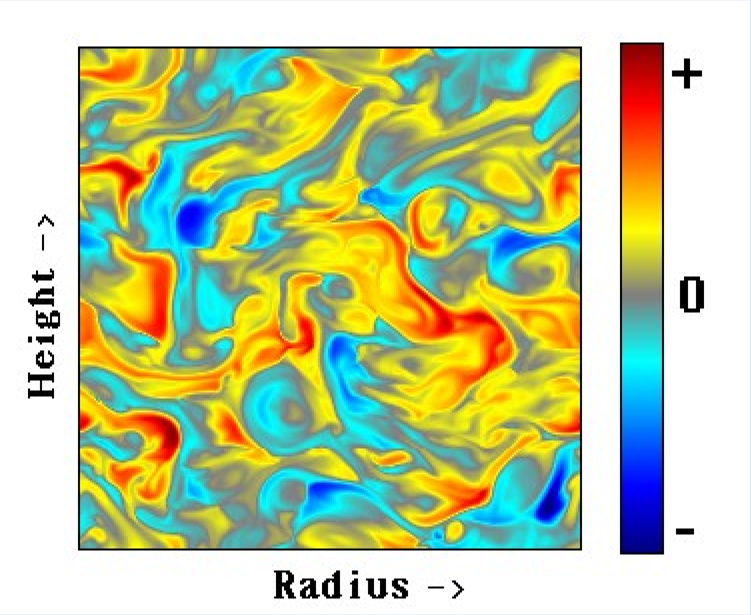
- The kinematic viscosity, according to Shakura & Sunyaev (1973) prescription, is

$$\nu = \alpha c_s H$$

with a constant α~0.1.

Magnitude of viscosity

- There is no reason to believe that viscosity is constant in time and throughout the disk.
- Probably, hydrodynamic mechanisms cannot produce sustained turbulence in differentailly rotating disks
- Balbus & Hawley (1991) studied magnetorotational instabilities (MRI) and found they are effective in driving turbulences



Magnitude of viscosity

- For Keplerian disk, we have $\Omega' = (3/2)\,\Omega$ and from hydrostatic equlibrium, $c_s^2 = P/\rho = H^2\Omega^2$
- Therefore, the shear stress is $T_{r\phi} = 3/2\,\alpha\,P$ and viscosity magnitude can be estimated as

$$\alpha = 2/3\ T_{r\phi}/P$$

The stress computed in the shearing-box MHD simulations consists of Maxwell (magnetic) and Reynolds (turbulent) parts. Recent simulatiuons show that consistent values of alpha are obtained with total pressure (Hirose et al. 2009; Jiang et al. 2013)

Break

Radial structure of accretion disk

- The mass conservation equation

$$R\frac{\partial \Sigma}{\partial t}+\frac{\partial}{\partial R}(R\Sigma V_R)=0$$

- Conservation of angular momentum

$$R\frac{\partial}{\partial t}(\Sigma R^2\Omega)+\frac{\partial}{\partial R}(R\Sigma V_R R^2\Omega)=\frac{1}{2\pi}\frac{\partial G}{\partial R}$$

- Here V_R is the radial 'drift' velocity in the gas, and Ω is the (Keplerian) angular velocity. G(R,t) is the viscous torque, given by the kinematic viscosity and Ω'.

Radial structure

- Combining these two equations to eliminate V_R, we get the nonlinear diffusion equation:

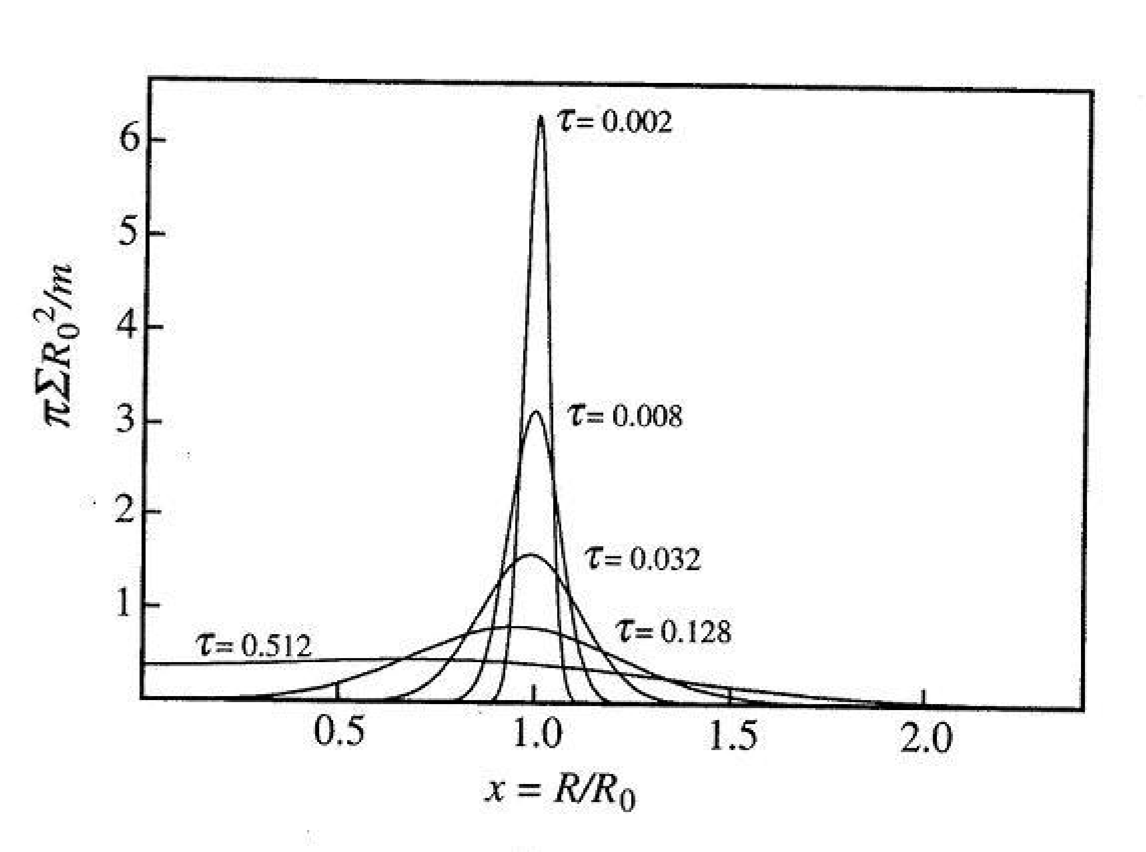
$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} [R^{1/2} \frac{\partial}{\partial R} [\nu \Sigma R^{1/2}]]$$

and the radial velocity is given by

$$V_R = - \frac{3}{\Sigma R^{1/2}} \frac{\partial}{\partial R} [\nu \Sigma R^{1/2}]$$

- If ν is constant, then we can solve this by separation of variables, substituting $y=2R^{1/2}$

Spreading of a ring



A ring of matter with mass m is initially placed in Keplerian orbit at R_0. Dimensionless time: $\tau = 12\,\nu\,t\,R_0^{-2}$

Viscous timescale

- Spreading of the original ring in radius occurs on typical timescale

$$t_{visc} \sim R^2/\nu \sim R/V_R$$

- The outer parts of disk move outwards, taking away the angular momentum of the inner parts, which move inwards, to the central star.
- The radius at which V_R changes sign, moves outwards
- After $t \gg t_{visc}$, almost all of the original mass will be accreted. All of the original angular momentum will be carried to very large radii, by a very small fraction of mass.

Steady disk

- External conditions may change on timescales much longer than t_{visc}. Therefore we can neglect time derivatives and

$$R\Sigma V_R = const = -\frac{1}{2\pi}\dot{M}$$

- From the angular momentum equation, we get

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1-\left(\frac{R_{star}}{R}\right)^{1/2}\right]$$

where the term in brackets comes from the constant, determined by inner boundary condition (it is not valid if the star has a strong magnetic field, or rotates much faster than $\Omega_K(R_*)$).

Steady disk

Energy flux through the faces of a steady disk:

$$D(R)=\frac{1}{2}\nu\Sigma(R\Omega')^2=\frac{3GM\dot{M}}{8\pi R^3}[1-(\frac{R_{star}}{R})^{1/2}]$$

and is independednt of viscosity (because we used conservation laws to eliminate it).

Structure in the vertical direction

- Hydrostatic equilibrium (given by the z-component of the Euler equation, with neglected velocity terms)

$$\frac{1}{\rho}\frac{\partial P}{\partial z}=\frac{\partial}{\partial z}\left[\frac{GM}{(R^2+z^2)^{1/2}}\right]$$

- For a thin disk, z<<R, H<<R, z~H, dP/dz ~ P/H
- The local Kepler velocity is highly supersonic: $H \approx c_s/\Omega^2$, so $c_s << (GM/R)^{1/2}$
- The radial drift velocity is subsonic, $V_R \sim \alpha\, c_s\, H/R << c_s$

Local vertical structure

- Vertical structure is decoupled from radial (treated as 1-d version of stellar structure)
- Vertical energy transport may be radiative or convective, depending on temperature gradient
- For radiative transport, the flux through z=const surface (in plane-parallel approximation) is

$$F(z) = -\frac{16\sigma T^3}{3\kappa\rho}\frac{\partial T}{\partial z}$$

  with the Rosseland-mean opacity $\kappa(\rho, T)$.
- The disk is optically thick if $\tau = \kappa\rho H = \kappa\Sigma \gg 1$.

Energy balance

- The energy balance requires:

F(z=H) - F(z=0)= D(R)

- If surface temperature is much smaller than central temperature, this will be

$$\frac{4\sigma T_c^4}{3\tau} = \frac{3GM\dot{M}}{8\pi R^3}\left[1-\left(\frac{R_{star}}{R}\right)^{1/2}\right]$$

- The closing relation is the equation of state, and in general the pressure is a sum of gas and radiation pressures:

$$P = \frac{k}{\mu m_p}\rho T_c + \frac{4\sigma}{3c} T_c^4$$

with mean molecular weight μ~1 for neutral hydrogen

Alpha prescription

- N. Shakura and R. Sunyaev (1973) introduced a very famous prescription for the magnitude of viscosity in accretion disk
- The ignorance about the real magnitude of kinematic viscosity is hidden in a scaling ”alpha” parameter: $\nu = \alpha\, c_s H$
- The magnitude of viscosity cannot be larger than the sound speed and disk thickness lenght scale: alpha must be less than 1.0. Otherwise not specified!

Steady state disk solutions

- $\Sigma = 5.2\ \alpha^{-4/5}\ dM_{16}^{\ 7/10}\ m_1^{\ 1/4}\ R_{10}^{\ -3/4}\ f^{\ 14/5}\quad [g\ cm^{-2}]$
- $H = 1.7 x 10^8\ \alpha^{-1/10}\ dM_{16}^{\ 3/20}\ m_1^{\ -3/8}\ R_{10}^{\ 9/8}\ f^{\ 3/5}\quad [cm]$
- $\rho = 3.1 x 10^{-8}\ \alpha^{-7/10}\ dM_{16}^{\ 11/20}\ m_1^{\ 5/8}\ R_{10}^{\ -15/8}\ f^{11/5}\ [g\ cm^{-3}]$
- $T_c = 1.4 x 10^4\ \alpha^{-1/5}\ dM_{16}^{\ 3/10}\ m_1^{\ 1/4}\ R_{10}^{\ -3/4}\ f^{\ 6/5}\ [K]$
- $\tau = 190\ \alpha^{-4/5}\ dM_{16}^{\ 1/5}\ f^{\ 4/5}$
- $\nu = 1.8 x 10^{14}\ \alpha^{\ 4/5}\ dM_{16}^{\ 3/10}\ m_1^{\ -1/4}\ R_{10}^{\ 3/4}\ f^{\ 6/5}\ [cm^2\ s^{-1}]$
- $V_R = 2.7 x 10^4\ \alpha^{\ 4/5}\ dM_{16}^{\ 3/10}\ m_1^{\ -1/4}\ R_{10}^{\ -1/4}\ f^{\ -14/5}\ [cm\ s^{-1}]$

S&S(1973). Assumed is μ=0.615 and radiation pressure neglected.

R_{10}=R/(10^{10} cm), m_1=M/M_{Sun}, dM_{16}=dM/(10^{16} g s^{-1})

Characteristic timescales

- Viscous timescale – on which the matter diffuses through the disc under the viscous torque:

$$t_{visc} \approx \frac{R^2}{\nu} \approx \frac{R}{V_R} \approx \frac{1}{\alpha} \left(\frac{H}{R}\right)^{-2} t_{dyn}$$

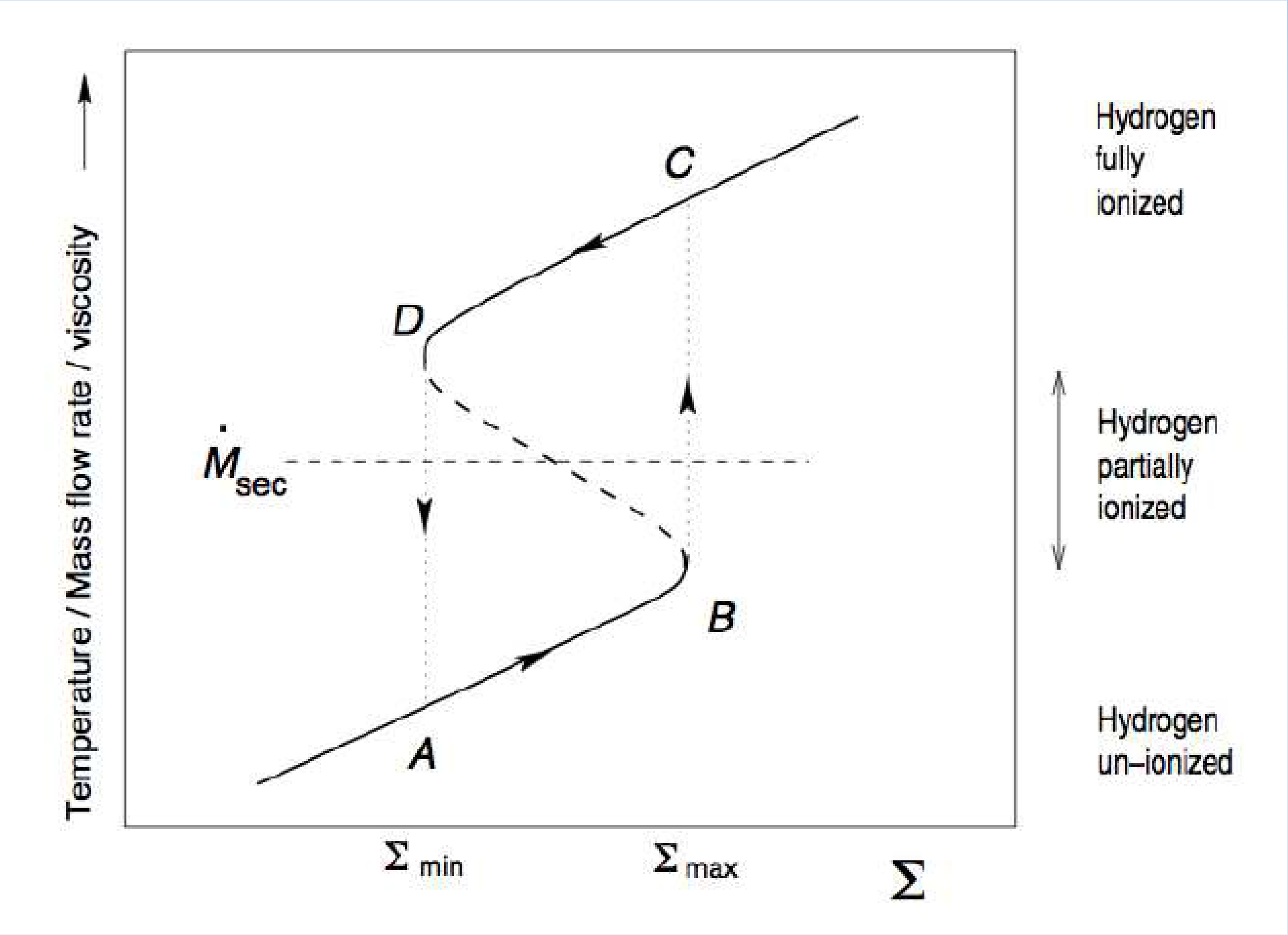
- Dynamical timescale, on which small inhomogeniteis in the disk arise, and the hydrostatic equlibrium in the vertical direction is established:

$$t_{dyn} \approx \frac{R}{V_\phi} = \frac{1}{\Omega_K}$$

- Thermal timescale, over which the thermal equilibrium is adjusted. It is the ratio of heat content to the dissipation rate (per unit disk area): $\boldsymbol{t_{th} \sim 1/\alpha\, t_{dyn} \ll t_{visc}}$

Energy balance

- Local solution is represented by a point in the (density, temperature) plane
- Positive slope means stable solution



Thermal instability

- If the energy balance is disturbed in the disk, any instability will grow in t_{th}. The surface density may be fixed on this timescale
- The vertical structure will respond to these changes on t_{dyn}, so it will be kept close to hydrostatic equilibrium.
- Instabilities arise when volume cooling rate Q^- can no longer cope with the volume heating rate $Q^+ \sim D/H$. When temperature T_c is increased by a small perturbation and Q^+ increases faster than Q^-, a steady state is impossible.
- Criterion: $\mathbf{d \log Q^- / d \log T_c < d \log Q^+ / d \log T_c}$

Viscous instability

- The disk on viscous timescale evolves much slower and maintains thermal and hydrostatic equilibrium
- Mass transfer rate depends on time at every radius, mass conservation equation is time dependent
- In diffusion equation, with $\mu = \nu\Sigma \sim$ Mdot $\sim$ T^4,

$$\frac{\partial}{\partial t}(\Delta \mu) = \frac{\partial \mu}{\partial \Sigma} \frac{3}{R} \frac{\partial}{\partial R}\left[R^{1/2} \frac{\partial}{\partial R}\left(R^{1/2} \Delta \mu\right)\right]$$

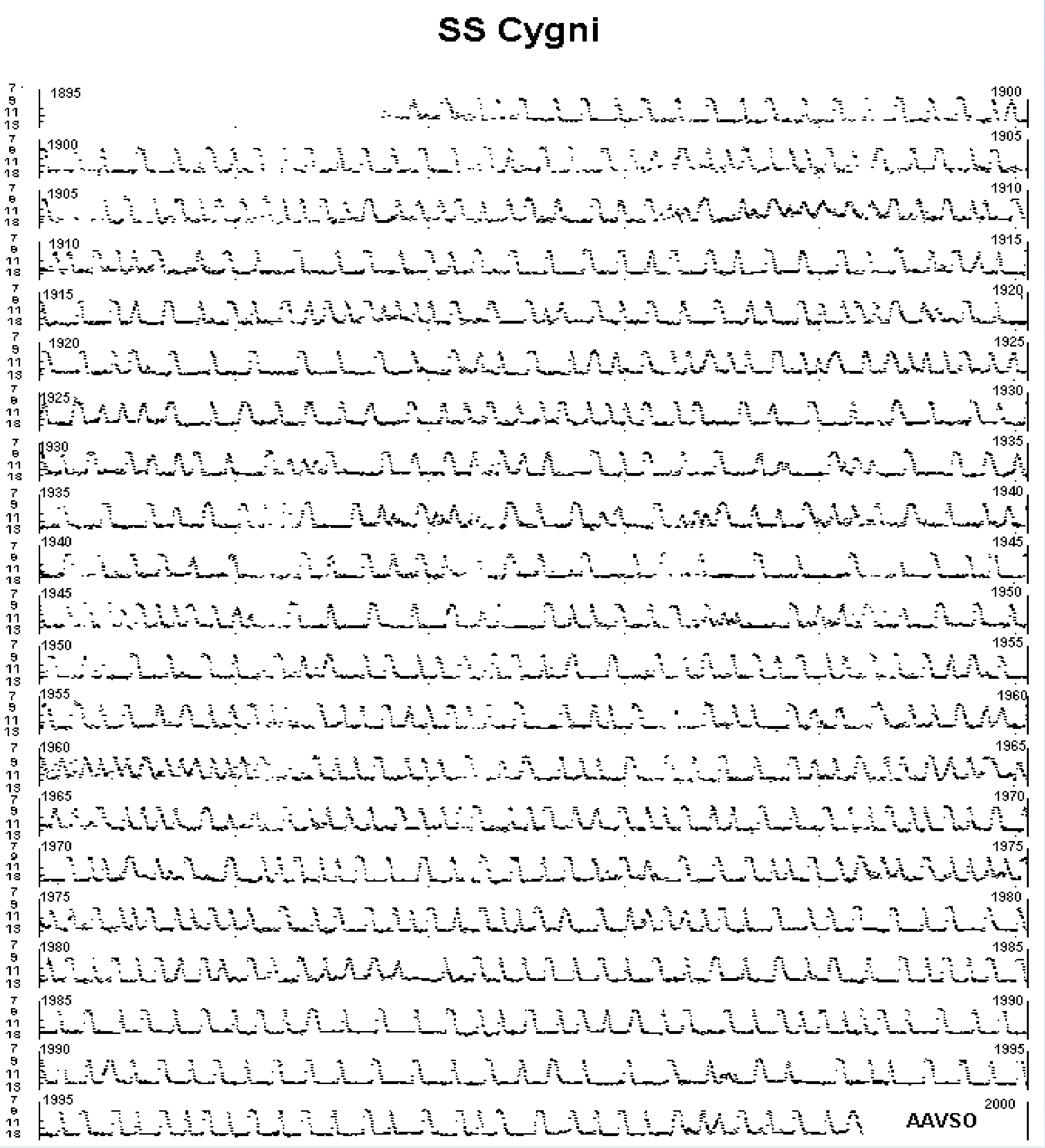
- if the coefficient is positive, then the perturbation will decay on the viscous timescale. For:

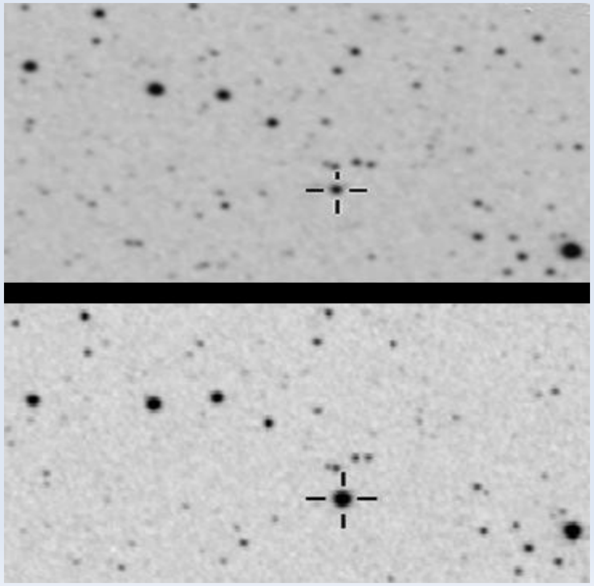
$$\frac{\partial \dot{M}}{\partial \Sigma} < 0$$

the local mass transfer rate will increase in response to decrease of density → matter will be removed from regions that are less dense

SS Cygni

- Discovered in 1896
- Prototype of Dwarf Nova class
- Outbursts every 7-8 weeks, rising from +12 to +8 mag, in timescale of 2 days





Next week

- Dwarf novae. Recurrent novae, cataclysmic variables.
- Instability of accretion disk due to partial hydrogen ionisation.
- Modeling of instability cycles.
- Another thermal-viscous instability process in the accretion disk: radiation pressure instability.