Modified viscosity in accretion disks

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Black holes surrounded by accretion disks are present in the Universe in different scales of masses, from microquasars, up to Active Galactic Nuclei. The basic theory of a geometrically thin, stationary accretion is based on α description which assumes the proportionality between non-diagonal stress tensor term and the total pressure. Domination of thermal pressure leads to thermal instability, which results in limit-cycle oscillations in sources like like GRS1915+105 and IGR J17091-3624. In our work we examined large grid of accretion disk models with generalized description of viscosity.

In general, the parameter range of the presence of limit-cycle oscillations and their amplitude can be reduced by magnetic field. We model its influence by extending the global code GLADIS to include the μ magnetic viscosity parameter.

We used this modelling to determine the mass of the intermediate mass black hole of HLX-1 and its accretion rate from the observed flares, detected by the Swift X-ray satellite.

Furthermore, we extend our model including atomic processes It partially reduced the instability of the disk in case of Active Galactic Nuclei.

1 Introduction

The accretion disks are composed form matter falling down onto a black hole or other kind of compact object (like Neutron Stars or White Dwarfs). In this paper we focus on the case of black hole accretion disks. The accretion disks are marked out by the characteristic spectrum, containing the thermal emission (being the blackbody radiation for the special radial temperature profile ($T \propto r^{-3/4}$). We can observe the accretion disks among all the black hole mass-scale, from the stellar-mass black holes, being results of the final evolution of the most massive stars (about 10 $M_{\odot}$) up to supermassive black hole inside Active Galactic Nuclei.

The necessary condition for the balance between gravitational attraction and emitted radiation pressure leads to distinction of special value of the luminosity called Eddington Luminosity. This value is proportional to the central object (Black Hole) mass $L_{\rm Edd} = 1.26 \times 10^{38} {\rm erg/s} \frac{M}{M_{\odot}}$ It results in the proportionality between the possible maximal luminosity of the accretion disk sources and the Black Hole Mass. For example (Altamirano et al., 2011a; Belloni et al., 2000) the luminosity of the most well-known stellar mass accretion disk sources in at the order of $10^{37} - 10^{38}$erg/s. The main examples of those sources are microquasars like GRS 1915+105, IGR J17091-3624, GX 339-4, the UltraLuminous X-ray source HLX-1 and Gigahertz Peak Quasars like 0108+388, 0710+439 (Wu et al., 2016). The temperature of those sources is high enough to emit X-ray radiation and be observed by the X-ray observatories.

2 Theory

2.1 Gas and radiation

The accretion disks are hot enough to contain a significant fraction of the radiation in their total pressure. Let us write the equation of state:

$$P = P_{\rm rad} + P_{\rm rad} \tag{1}$$

$$P_{\rm gas} = a\rho T \tag{2}$$

$$P_{\rm rad} = bT^4 \tag{3}$$

Where in Eq. (2) $a = \frac{k_B}{m_H}$ where k_B - Boltzmann constant, m_H - proton mass, $a = \frac{4\sigma_B}{3c}$ and in Eq. (3) σ_B - Stefan-Boltzmann constant, c - speed of light. The first term of the Eq. (1) - gas pressure, second term - total pressure. Internal energy:

$$u = \frac{3}{2}aT + 3\frac{bT^4}{\rho} \tag{4}$$

Defining $\beta = \frac{P_{\rm gas}}{P}$, we get following formula for the differential density of energy

$$du = \frac{P}{\rho}((12 - 10.5\beta)d\log T - (4 - 3\beta)d\log\rho \tag{5}$$

2.2 Model assumptions

In our model we assume set of hydrodynamics equations in a rotating frame under some assumption which reduce the number of the equations (Janiuk, Czerny & Siemiginowska , 2002). We describe the disk using following local parameters density ρ, temperature T, thickness H, surface density $\Sigma = \rho H$. Another assumption are axial symmetry of the disk, local vertical hydrostatic equilibrium and Keplerian angular velocity. In our equations we reduce heat diffusion in the radial direction. The equations are as follows:

$$\frac{\partial\Sigma}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\Sigma v_r) \tag{6}$$

where radial velocity $v_r = -\frac{3}{\Sigma}r^{-1/2}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2})$

$$\frac{\partial\ln T}{\partial t} + v_r\frac{\partial\ln T}{\partial\ln r} = \frac{4-3\beta}{12-10.5\beta}(\frac{\partial\ln T}{\partial t} + v_r\frac{\partial\ln\rho}{\partial\ln r})\frac{Q_+ - Q_-}{12-10.5\beta} \tag{7}$$

The heating rate Q_+ and viscosity ν can be derived form non-diagonal terms of stress tensor $T_{r\phi}$ as follows: $\nu = \frac{T_{r\phi}}{\rho\nu\frac{\partial\Omega}{\partial r}}$,$Q_+ = 1.25T_{r\phi}r\frac{\partial\Omega}{\partial r}$. Let κ be the atomic opacity expressed in cm^2 g^{-1}. In our model we assume the geometrically thin and optically thick disk (value of non-dimensional thickness τ is large in comparison to 1) so the cooling function is expressed as follows:

$$Q_- = \frac{c}{\tau}P_{\rm rad} = \frac{c}{\kappa\Sigma}T^4 \tag{8}$$

2.3 Disk viscosity model

Shakura & Sunyaev (1973) proposed that the stress tensor in the accretion disk is proportional to the total pressure with constant α, which provides the effective description of the turbulent dissipation in accretion disks

$$T_{r\phi} = \alpha P \tag{9}$$

Later, Lightman & Eardley (1974) marked that this model is self-inconsistent and may lead to the disintegration of the disk. (Sakimoto & Coroniti, 1981) proposed

$$T_{r\phi} = \alpha P_{\rm gas} \tag{10}$$

Then, Taam & Lin (1984) proposed an intermediate prescription which avoids problems from both Eqs. (9,10):

$$T_{r\phi} = \alpha \sqrt{P P_{\rm gas}} \tag{11}$$

Basing on Szuszkiewicz (1990), we propose more general model with parameter μ

$$T_{r\phi} = \alpha P^{\mu} P_{\rm gas}^{1-\mu} \tag{12}$$

2.4 Timescales

In accretion disks we distinct different timescales - dynamical (t_d) - corresponding to the sound waves, thermal (t_{th}) - corresponding to the local thermal processes, and viscous (t_{visc}) corresponding to the global evolution of the disk. The orders of magnitude of those scales are as follows:$t_d = \sqrt{r^3/GM} \approx 10^{-5}\frac{M}{M_\odot}$s, $t_{th} = \alpha^{-1}\sqrt{r^3/GM} \approx 2 \times 10^{-3}\frac{M}{M_\odot}$s and $t_{visc} = \alpha^{-1}\sqrt{r^3/GM}(\frac{R}{H})^2 \approx 0.2 - 200\frac{M}{M_\odot}$s In above model, those scales are defined by the disk geometry and α parameter.

2.5 Thermal stability analysis

In our model we average the disk behaviour at the dynamical timescales and focus on the thermal and viscous timescales. We have the condition: Stability condition can be written in form:

$$\frac{d \log Q_+}{d \log T} > \frac{d \log Q_-}{d \log T} \tag{13}$$

Applying (13) to the heating function defined by (12) we get:

$$\frac{d \log Q_+}{d \log T} = 1 + \frac{7}{2}\mu \frac{1-\beta}{1+\beta} \tag{14}$$

$$\beta < \frac{7\mu - 3}{7\mu + 3} \tag{15}$$

Instability occurs if $\mu > 7/3$; μ can model stabilizing influence of the magnetic field. The thermal instability combined with stabilizing advective processes leads to the limit-cycle oscillations which will be presented in the next Section.

3 Model Results

In our work we used the time-dependent code GLADIS using Eqs. (6,7). We parameterized our input with four parameters -black hole mass M, Eddington ratio $\dot{m} = \frac{\dot{M}}{\eta c^2}$ [1], α and μ. Our output was parameterized as follows - average luminosity L, period between to following flares P, non-dimensional flare-duration-to-period-ratio Δ and non-dimensional amplitude A [2]. We did 3 grids of models for the cases of stellar mass ($10 M_\odot$), intermediate-mass ($3 \times 10^4 M_\odot$) and supermassive ($10^8 M_\odot$) black holes. We ran our models for fixed $\alpha = 0.02$ and different values of $\dot{m}$ and μ.

3.1 Results form grid

We get three most important correlations - **duration - luminosity** (Fig. 1), μ **- flare width** (Fig. 3, Eq. 16) and **period - amplitude** (Fig. 2, Eq. 17)

$$\log \Delta = \frac{1.9 + 1.2 \log M}{\mu - 3/7} \tag{16}$$

$$\log P \approx 0.83 \log A + 1.15 \log M + 0.40 \tag{17}$$

3.2 Application - HLX-1 mass determination

From Eqs. 16 and 17 we can do the inverse procedure and determine the mass and μ parameter directly form the observations:

$$M = 0.45 P^{0.87} A^{-0.72} (\frac{\alpha}{0.02})^{1.88} \tag{18}$$

Determination of μ magnetization parameter:

$$\mu = \frac{3}{7} + \frac{-\log \Delta + 0.87 \log(\frac{\alpha}{0.02})}{1.49 + 1.04 \log P - 0.864 \log A} \tag{19}$$

From Eq. 18, basing on the X-ray observations (Wu et al., 2016), we can determine the mass of HLX-1 intermediate mass black hole. HLX-1 is the best known case of a ULX being an IMBH candidate, which has undergone six outbursts spread in time over several years with an average period of about 400 days, a duration of about 30-60 day (Wu et al., 2016). Finally, we determined the HLX-1 mass at the level of $M = 1.9 \times 10^5 M_\odot$ and accretion rate at the level of Accretion rate $0.09 - 0.18$ in Eddington units (assuming correction K between 10 and 20 , see (Grzędzielski et al., 2017a) for more detailed discussion).

4 Atomic opacities and their influence on disk stability

In our previous model we assumed κ being constant (Thomson opacity) now we include atomic opacities known from tables (Alexander et al. (1983),Seaton et al. (1994)). According to some last results (Jiang et al., 2016) the so-called *Iron Opacity*

[1] $\eta = 1/16$ is accretion efficiency

[2] Defined as maximum to minimum luminosity ratio

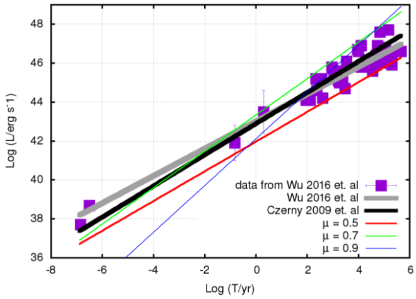

Fig. 1: Correlation between the bolometric luminosity and the outburst duration for different-scale BHs.

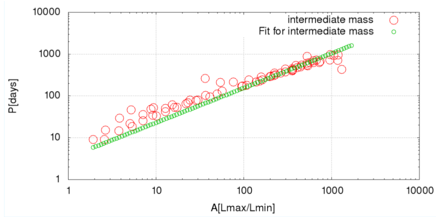

Fig. 2: Correlation between period, amplitude and black hole mass

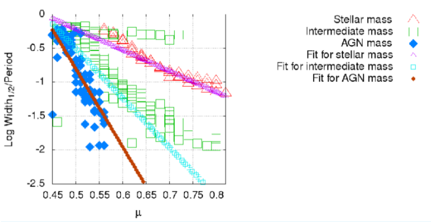

Fig. 3: Correlation between μ parameter and lightcurve shape

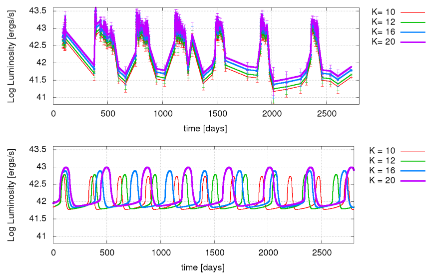

Fig. 4: Mass determination for HLX-1 for different corrections K

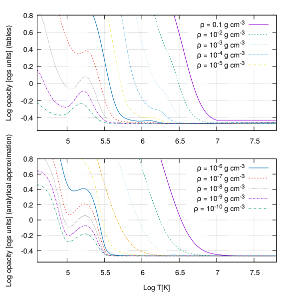

Fig. 5: Total opacity function for different temperatures and densities (upper panel), and our fit (lower panel)

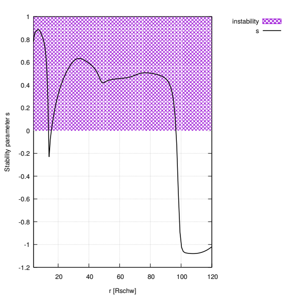

Fig. 6: Stability parameter for different r $M = 5 \times 10^8 M_\odot$ and $\dot{m} = 0.03$

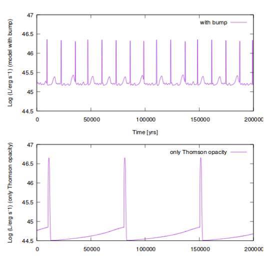

Fig. 7: Stability parameter for different r $M = 5 \times 10^8 M_\odot$ and $\dot{m} = 0.03$

Bump has big influence on the disk stability We model the atomic opacity function by power-law tail + gaussian bump. We can define stability parameter $-s = \frac{d\log Q_+}{d\log T} - \frac{d\log Q_-}{d\log T} == -3 + 7\mu\frac{1-\beta}{1+\beta} - \frac{\partial\log\kappa}{\partial\log T} + \frac{4-3\beta}{1+\beta}\frac{\partial\log\kappa}{\partial\log\rho}$ For $s > 0$ disk is locally thermally stable, for $s < 0$ locally thermally unstable. We see from Fig. 6 that in the disk two unstable zones are separated by the stable ring. That ring corresponds to the stabilizing influence of the Iron Opacity Bump, although it does not suppress the oscillations, only stabilizes them (see Figure 7).

5 Conclusions

Generalized model with μ parameter makes possible determination of the BH mass. Our computations are an an argument for radiation pressure instability as a source of outburst of many X-ray binaries like GRS 1915, IGR J17091 (Grzędzielski et al. (2017a)) The μ-model is a way for effective description of a global stabilizing magnetic field. We also shown that atomic opacities can play some role in Active Galactic Nuclei, but stabilization is only partial ((Grzędzielski et al., 2017b) in contradiction to Jiang et al. (2016) short time shearing box simulation).

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