Theory of thermal instabilites in accretion disks and its applications to the X-ray sources in all scales

Mikołaj Grzedzielski

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The accretion disks

- matter falling down onto central object (black hole, neutron star, star)
- Common around the black holes (and another objects in all scalles of masses
- Characteristic disk spectrum (we can detect the disk from the X-ray spectral continuum)
- The Eddington limit connect their maximal luminosity in central mass

$$L_{Edd} = 1.26 \times 10^{38} \frac{M}{M_{\odot}} \quad (1)$$

- examples - GRS1915, IGR J17091, GX 339-4 (microquasars) , HLX-1 (ULX), 0108+388, 0710+439 (Gigahertz Peak Quasars)

Basic hydrodynamics equations in rotating frame

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho v_r)}{\partial r} + \frac{\partial(\rho v_z)}{\partial z} + \frac{1}{r}\frac{\partial(\rho\Omega)}{\partial\phi} = 0 \quad (2)$$

$$\rho(\frac{\partial}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \Omega\frac{\partial v_r}{\partial\phi} - \Omega^2 r + v_z\frac{\partial v_r}{\partial z}) = -\frac{\partial p}{\partial r} - \frac{\partial\tau_{r\phi}}{\partial\phi} - \frac{\partial\tau_{rz}}{\partial z} - \rho\Omega_K^2 r \quad (3)$$

$$\rho(\frac{\partial}{\partial t} + v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} + \Omega\frac{\partial v_z}{\partial\phi}) = -\frac{\partial p}{\partial z} - \frac{\partial\tau_{z\phi}}{\partial\phi} - \frac{\partial\tau_{zr}}{\partial r} - \rho\Omega_K^2(\frac{Rz}{\sqrt{R^2+z^2}}) \quad (4)$$

$$\rho(\frac{\partial(\Omega r^2)}{\partial t} + v_r\frac{\partial(\Omega r^2)}{\partial r} + \Omega\frac{\partial(\Omega r^2)}{\partial\phi} + 2v_r\Omega r + v_z\frac{\partial(\Omega r^2)}{\partial z}) = -\frac{\partial p}{\partial\phi} - \frac{\partial\tau_{\phi r}}{\partial r} - \frac{\partial\tau_{\phi z}}{\partial z} \quad (5)$$

$$T[\frac{\partial}{\partial t} + (v_r\frac{\partial}{\partial r} + v_z\frac{\partial}{\partial z} + \Omega\frac{\partial}{\partial\phi})]S = q_+ - q_- \quad (6)$$

Thermodynamics of gas - radiation mixture

Gas + radiation

$$P = a\rho T + bT^4 \tag{7}$$

Where $a = \frac{k_b}{m_H}$ k_b - Boltzmann constant, m_H - proton mass, and $a = \frac{\sigma_B}{3c}$ where σ_B - Stefan-Boltzmann constant, c - speed of light. The first term of the 7 correspond to the gas pressure, the second one- to the total pressure.

$$u = \frac{3}{2}aT + 3\frac{bT^4}{\rho} \tag{8}$$

Defining $\beta = \frac{P_gas}{P}$, we get following formula on the differential density of energy

$$du = \frac{P}{\rho}((12 - 10.5\beta)d\log T - (3 - 3\beta)d\log\rho) \tag{9}$$

From the thermodynamic rules

$$TdS = du - \frac{p}{\rho^2} d\rho \tag{10}$$

We get the formular for the entropy:

$$TdS = \frac{P}{\rho}((12 - 10.5\beta) d \log T - (4 - 3\beta) d \log \rho) \tag{11}$$

The equation (6) gains the form:

$$(12-10.5\beta)[\frac{\partial \log T}{\partial t} + v_r \frac{\partial \log T}{\partial r} + v_z \frac{\partial \log T}{\partial z} + \Omega \frac{\partial \log T}{\partial \phi}] + (4-3\beta)[\frac{\partial \log \rho}{\partial t} + v_r \frac{\partial \log \rho}{\partial r} + v_z \frac{\partial \log \rho}{\partial z} + \Omega \frac{\partial \log \rho}{\partial \phi}] = q_+ - q_- \tag{12}$$

From Eq.(12), putting $\rho = \frac{\Sigma}{H}$, we get the final equation:

$$\frac{\partial \log T}{\partial t} + v\frac{\partial \log T}{\partial r} = \frac{4-3\beta}{12-10.5\beta}\left(\frac{\partial \log \Sigma}{\partial t} - \frac{\partial \log H}{\partial t} + v\frac{\partial \log \Sigma}{\partial r} - v\frac{\partial \log H}{\partial r}\right) + \frac{Q_+ - Q_-}{(12-10.5\beta)PH} \quad (13)$$

Which is equivalent to Eq. (33) in paper [2] which we use in our work if only we assume that disk is not strongly deformed (logarythmic derivative of H ($v_r\frac{\partial \log H}{\partial r}$) is small in comparison to logarythmic derivative of Σ ($v_r\frac{\partial \log \Sigma}{\partial r}$)) .

Thin disk hydrodynamics - assumptions

The case of geometrically-thin, optically think accretion disk is based on some assumptions:

- Disk is optically thick, which leads to the surface emission function (amount of energy per unit of time per surface unit) as follows:

$$Q_{-} = \frac{4\sigma_B T^4}{3\kappa\Sigma} \quad (14)$$

  where σ_B is Stefan-Boltzmann constant, κ - scattering opacity between photons and matter, and Σ - is the surface density of the the matter in disk.

Thin disk hydrodynamics - assumptions

- The viscous transfer of the momentum is limited to the disk plane

$$\tau_{rz} = \tau_{zr} = \tau_{\phi z} = \tau_{z\phi} = 0 \qquad (15)$$

- we assume the Shakura-Sunyaev α prescription for the stress tensor. We assume the general input of the

$$\tau_{r\phi} = \alpha P^{\mu} P_{\mathrm{gas}}^{1-\mu} \qquad (16)$$

- The disk is thin - $H << r$ - the thickness is many times smaller than the distant from the centre

Timescales for BH accretion disks

- dynamical timescale $t_d = \sqrt{r^3/GM} \approx 10^{-5} \frac{M}{M_\odot}$s
- thermal timescale $t_{th} = \alpha^{-1}\sqrt{r^3/GM} \approx 2 \times 10^{-3} \frac{M}{M_\odot}$s
- viscous timescale $t_{visc} = \alpha^{-1}\sqrt{r^3/GM}(\frac{H}{R})^2 \approx 0.2 - 200 \frac{M}{M_\odot}$s

Time-dependent model

Our model consists of two equations describing evolution of the disk in the thermal and viscous timescales:

$$\frac{\partial \Xi}{\partial t} = \frac{12}{y^2}\frac{\partial^2}{\partial y^2}(\Xi \nu), \qquad (17)$$

where $y = 2r^{1/2}$ and $\Xi = 2r^{1/2}\Sigma$, and

$$\frac{\partial \log T}{\partial t} + v\frac{\partial \log T}{\partial r} = \frac{4-3\beta}{12-10.5\beta}\left(\frac{\partial \log \Sigma}{\partial t} - \frac{\partial \log H}{\partial t} + v\frac{\partial \log \Sigma}{\partial r}\right) + \frac{Q_+ - Q_-}{(12-10.5\beta)PH} \qquad (18)$$

Thermal stability analysis

The condition:

$$\frac{d\log Q_+}{d\log T} > \frac{d\log Q_-}{d\log T}. \tag{19}$$

For thermal timescale: $\Sigma = const$. Heating:

$$Q+ = \frac{3}{2}\alpha P^{\mu} P_{\rm gas}^{1-\mu} H\Omega. \tag{20}$$

Hydrostatic equilibrum:$H = P/(C_3\Sigma\Omega)$, $x = \frac{P_{\rm gas}}{P_{\rm rad}} + 0.5$, we can rewrite Eq. (20) as:

$$Q_+ = \frac{3}{2C_3\Sigma\Omega} P^{1+\mu} P_{\rm gas}^{1-\mu} = \frac{3}{2C_3\Sigma\Omega} P_{\rm rad}(x+1/2)^{1+\mu}(x-1/2)^{1-\mu}. \tag{21}$$

Then, if we assume a constant Σ regime, we have:

$$\frac{dx}{dT} = -\frac{7}{2}\frac{(x+1/2)(x-1/2)}{xT} \tag{22}$$

Thermal stability analysis -results

$$\frac{d\log Q_+}{d\log T} = 1 + \frac{7}{2}\frac{1-\beta}{1+\beta} \tag{23}$$

where $\beta = \frac{P_{gas}}{P_{tot}} = \frac{x-1/2}{x+1/2}$. Finally, from Eq. (14) and Eq. (19), we have:

$$\frac{d\log Q_+}{d\log T} > 4, \tag{24}$$

which is fullfilled if the condition:

$$\beta < \frac{7\mu - 3}{7\mu + 3} \tag{25}$$

is satisfied [3]. This gives the necessary condition for the instability for the case of μ-model, so that the instability occurs only if $\mu > 3/7$.

Thermal stability analysis

Strong magnetic field can stabilize the disk [8] We assume magnetic contribution to the pressure

$$P = P_{\rm rad} + P_{\rm gas} + P_{\rm mag}. \tag{26}$$

Let us define the disk magnetization coefficient $\beta' = \frac{P_{mag}}{P_{tot}}$. We put the formula (26) into the Shakura-Sunyaev stress-energy tensor (i.e., for $\mu = 0$ in Eq. 21), and then we get:

$$\frac{d \log Q_+}{d \log T} = 8(1 - \beta'). \tag{27}$$

$\beta' > \frac{1}{2}$ - disk is stable $\frac{d \log Q_+}{d \log T} \leq \frac{d \log Q_-}{d \log T}$. From the formula (25) we can connect β' and μ as follows:

$$\mu = 1 - \frac{8}{7}\beta'. \tag{28}$$

β is equal to the magnetization parameter - the μ prescription may be connected

magnetization

The code

We use the 1.5 D time-depenedent code using Eqs. (17) and (18)
Input: $M, \dot{m}, \alpha, \mu$
Output: L, P , A, Δ

Period, amplitude and mass - model results - formulae

$$\log P[s] \approx 0.83 \log A + 1.15 \log M + 0.40 \quad (29)$$

Typical stellar masses:

$$\log P[s] \approx 0.83 \log A + 1.15 \log M/(10) + 1.55 \quad (30)$$

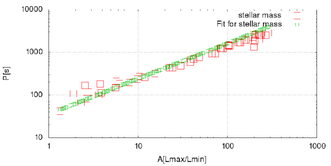
Typical IMBH masses:

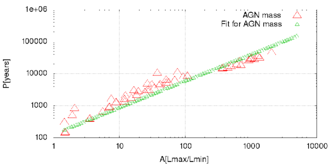
$$\log P[days] \approx 0.83 \log A + 1.15 \log M/(3 \times 10^4) + 0.53 \quad (31)$$

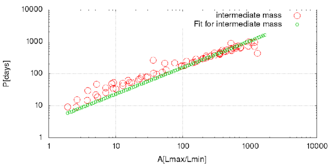
Typical AGN masses:

$$logP[yrs] \approx 0.83 \log A + 1.15 \log M/(10^8) + 2.1 \quad (32)$$

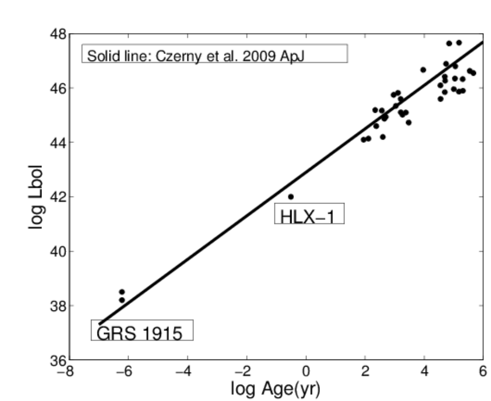
Period, amplitude and mass - model results - figures







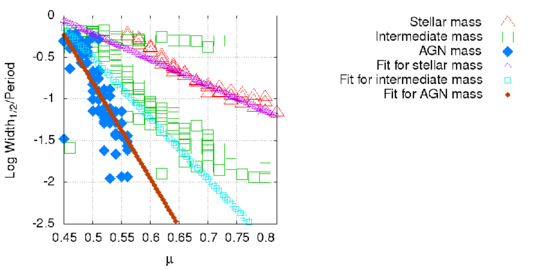
Results from previous works (like Czerny 2009 et. al)



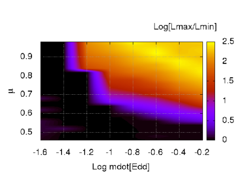
Width and magnetization

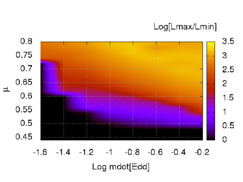
We have strong correlation between μ parameter and shape:

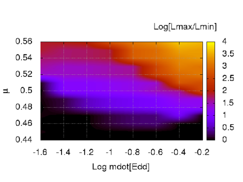
$$-\log \Delta = \frac{1.9 + 1.2 \log M}{\mu - 3/7} \tag{33}$$



Amplitude maps

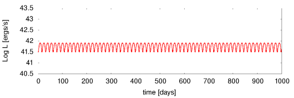


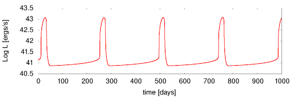




Nonlinearity in the lightcurves - flickering and outbursts

Left: comparison between ligthcurves for different values of μ
Right : comparison between duty cycles





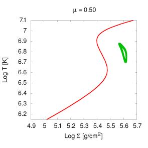


Figure : Flickering duty cycle

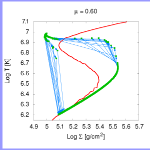
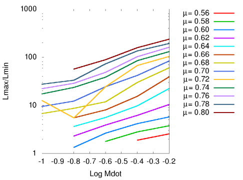
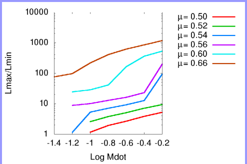
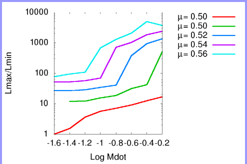


Figure : Burst duty cycle

Nonlinearity in the lightcurves - accretion rate and amplitude

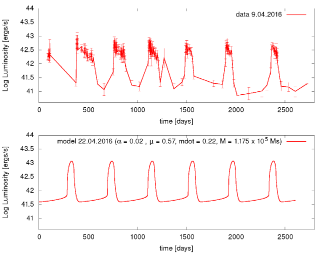






HLX-1 mass determination

- Ultraluminous X-ray source in ESO 243-49
- outbursts once per 400 days
- Intermediate-mass black hole candidate



Microquasar modeling

We determine μ and M from the

source	OBSID	State	P	A	Δ	M ($M_\odot$)	μ
IGR	ν_I	ν	45s	2.5	0.15	6.38	0.717
IGR	ρ_{IA}	ρ	30s	3.5	0.3	3.52	0.634
IGR	ρ_{IB}	ρ	30s	4	0.4	3.198	0.589
GRS	ν_G	ν	45s	4	0.1	8.31	0.763
GRS	ρ_{GA}	ρ	45s	5	0.25	3.77	0.661
GRS	ρ_{GB}	ρ	45s	4.5	0.4	6.38	0.583
HLX	-	-	400d	2.5	0.14	1.88×10^5	0.534
AGN	-	-	10^5y	100	0.1	1.6×10^8	0.515

Table : IGR = IGR J17091, GRS = GRS1915, HLX = HLX-1.The OBSIDs: $\nu_I = 96420-01-05-00$, $\rho_{IA} = 96420-01-06-00$, $\rho_{IB} = 96420-01-07-00$,$\nu_G = 10408-01-40-00$, $\rho_{GA} = 20402-01-34-00$ and $\rho_{GB} = 93791-01-02-00$.

Active Galactic Nuclei

- indirect methods
- for ensemble of sources $\frac{\partial N}{\partial L} \propto \dot{L}^{-1}$ (for given $M, \alpha, \dot{m}, \mu$)
- we know that the age is corellated with the bolometric luminosities (Wu et al. 2009) [5]
- Changing-look AGNs (like IC751) [7]

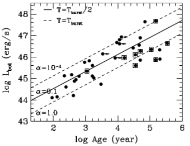


Figure 2. Correlation between ages and bolometric luminosities for young radio galaxies. The dashed lines are theoretical predictions from the disk instability model for L_{bol}–T_{burst} with $\alpha = 10^{-4}$ (upper) and 1 (lower), respectively, while the solid line is for L_{bol}–$T_{burst}/2$ with $\alpha = 0.1$. The bolometric luminosities of CSS sources calculated from [O III] luminosities are marked with extra open squares.

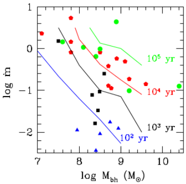
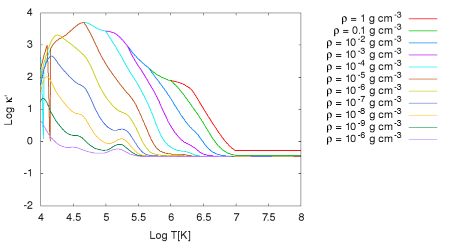


Figure 3. Comparison of the ages for young radio galaxies with the theoretical durations of outbursts triggered by disk instability in the M_{BH}–$\dot{m}$ plane. The triangles, squares, pentagons, and circles correspond to the age in bins of [0–5 × 10^2], [5 × 10^2–5 × 10^3], [5 × 10^3–5 × 10^4], and [5 × 10^4–5 × 10^5] yr, respectively. The solid lines are the half of theoretical outburst durations predicted by the disk instability model with $\alpha = 0.1$, which are con-

Atomic opacities

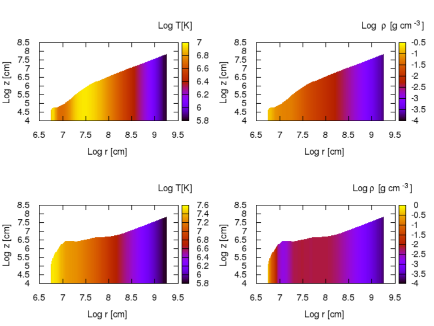


- Negative derivative stabilize the disk

$$\frac{d \log Q_-}{d \log T} = 4 - \frac{d \log \kappa}{d \log T} \quad (34)$$

The microquasars

- minimal stable orbit (changing caused by the BH spin) - can affect luminosity
- for positive spin source should be more luminous and vary faster
- for retrograde spin source - less luminous



Conclusions

- Global magnetic field stabilizes the disk
- We have a tool for mass measurement for new BHs with accretion disks
- Universal $\mu \approx 0.5 - 0.6$
- The method is applicable to objects in all scales

Thank you for attention

- *The universal "heartbeat" oscillations in black hole systems accross the mass-scale* Wu et. al (2016) https://arxiv.org/abs/1610.04402
- *Modified viscosity in accretion disks. Application to Galactic black hole binaries, intermediate mass black holes and AGN* Grzedzielski et. al (2016) https://arxiv.org/abs/1609.09322

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