Theory of thermal instabilites in accretion disks and its applications to the X-ray sources in all scales

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November 2016

Accretion disk model

The accretion disks

- matter falling down onto central object (black hole, neutron star, star)
- Common around the black holes (and another objects in all scalles of masses
- Characteristic disk spectrum (we can detect the disk from the X-ray spectral continuum)
- The Eddington limit connect their maximal luminosity in central mass

$$L_{Edd} = 1.26 \times 10^{38} \frac{M}{M_\odot} \quad (1)$$

- examples - GRS1915, IGR J17091, GX 339-4 (microquasars) , HLX-1 (ULX), 0108+388, 0710+439 (Gigahertz Peak Quasars)

Gas + radiation

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$$P = a\rho T + bT^4 \quad (2)$$

Where $a = \frac{k_b}{m_H}$ k_b - Boltzmann constant, m_H - proton mass, and $a = \frac{\sigma_B}{3c}$ where σ_B - Stefan-Boltzmann constant, c - speed of light. The first term of the 2 correspond to the gas pressure, the second one- to the total pressure.

$$u = \frac{3}{2}aT + 3\frac{bT^4}{\rho} \quad (3)$$

Defining $\beta = \frac{P_gas}{P}$, we get following formula on the differential density of energy

$$du = \frac{P}{\rho}((12 - 10.5\beta) d \log T - (3 - 3\beta) d \log \rho) \quad (4)$$

Entropy equation

From the thermodynamic rules

$$TdS = du - \frac{p}{\rho^2} d\rho \quad (5)$$

We get the formular for the entropy:

$$TdS = \frac{P}{\rho}((12 - 10.5\beta) d \log T - (4 - 3\beta) d \log \rho) \quad (6)$$

Equation of energy $(T[\frac{\partial}{\partial t} + (v_r \frac{\partial}{\partial r} + v_z \frac{\partial}{\partial z} + \Omega \frac{\partial}{\partial \phi})]S = q_+ - q_-)$ gains the form:

$$(12 - 10.5\beta)[\frac{\partial \log T}{\partial t} + v_r \frac{\partial \log T}{\partial r} + v_z \frac{\partial \log T}{\partial z} + \Omega \frac{\partial \log T}{\partial \phi}] + (4 - 3\beta)[\frac{\partial \log \rho}{\partial t} + v_r \frac{\partial \log \rho}{\partial r} + v_z \frac{\partial \log \rho}{\partial z} + \Omega \frac{\partial \log \rho}{\partial \phi}] = q_+ - q_- \quad (7)$$

From Eq.(7), putting $\rho = \frac{\Sigma}{H}$, we get the final equation:

$$\frac{\partial \log T}{\partial t} + v \frac{\partial \log T}{\partial r} = \frac{4 - 3\beta}{12 - 10.5\beta}(\frac{\partial \log \Sigma}{\partial t} - \frac{\partial \log H}{\partial t} + v \frac{\partial \log \Sigma}{\partial r} - v \frac{\partial \log H}{\partial r}) + \frac{Q_+ - Q_-}{(12 - 10.5\beta)PH} \quad (8)$$

Which is equivalent to Eq. (33) in paper [3] which we use in our work if only we assume that disk is not strongly deformed (logarythmic derivative of H $(v_r \frac{\partial \log H}{\partial r})$ is small in comparison to logarythmic derivative of Σ $(v_r \frac{\partial \log \Sigma}{\partial r})$) .

Disk theory assumptions

The case of geometrically-thin, optically think accretion disk is based on some assumptions:

- Disk is optically thick, which leads to the surface emission function (amount of energy per unit of time per surface unit) as follows:

$$Q_- = \frac{4\sigma_B T^4}{3\kappa\Sigma} \quad (9)$$

where σ_B is Stefan-Boltzmann constant, κ - scattering opacity between photons and matter, and Σ - is the surface density of the the matter in disk.

- The viscous transfer of the momentum is limited to the disk plane

$$\tau_{rz} = \tau_{zr} = \tau_{\phi z} = \tau_{z\phi} = 0 \quad (10)$$

- we assume the Shakura-Sunyaev α prescription for the stress tensor. We assume the general input of the

$$\tau_{r\phi} = \alpha P^\mu P_{\text{gas}}^{1-\mu} \quad (11)$$

- The disk is thin - $H << r$ - the thickness is many times smaller than the distant from the centre
- Hydrostatic equilibrum and axial symmetry limits number of independent parameters to 2

Timescales in accretion disks

- dynamical timescale $t_d = \sqrt{r^3/GM} \approx 10^{-5} \frac{M}{M_\odot}$s
- thermal timescale $t_{th} = \alpha^{-1}\sqrt{r^3/GM} \approx 2 \times 10^{-3} \frac{M}{M_\odot}$s
- viscous timescale $t_{visc} = \alpha^{-1}\sqrt{r^3/GM}(\frac{H}{R})^2 \approx 0.2 - 200 \frac{M}{M_\odot}$s

Model equations:

Our model consists of two equations describing evolution of the disk in the thermal and viscous timescales:

$$\frac{\partial \Xi}{\partial t} = \frac{12}{y^2} \frac{\partial^2}{\partial y^2}(\Xi \nu), \quad (12)$$

where $y = 2r^{1/2}$ and $\Xi = 2r^{1/2}\Sigma$, and

$$\frac{\partial \log T}{\partial t} + v \frac{\partial \log T}{\partial r} = q_{adv} + \frac{Q_+ - Q_-}{(12 - 10.5\beta)PH} \quad (13)$$

where

$$q_{adv} = \frac{4 - 3\beta}{12 - 10.5\beta}(\frac{\partial \log \Sigma}{\partial t} - \frac{\partial \log H}{\partial t} + v \frac{\partial \log \Sigma}{\partial r}) \quad (14)$$

Local stability analysis

The condition:

$$\frac{d \log Q_+}{d \log T} > \frac{d \log Q_-}{d \log T}. \quad (15)$$

For thermal timescale: $\Sigma = const$. Heating:

$$Q_+ = const \times \tau_{r\phi} \frac{\partial \Omega}{\partial r} H \quad (16)$$

$$Q_+ = const \times \frac{3}{2} \alpha P^\mu P_{\text{gas}}^{1-\mu} H\Omega. \quad (17)$$

Hydrostatic equilibrum:$H = const \times P/(\Sigma\Omega)$, we can rewrite Eq. (17) as: Then, if we assume a constant Σ regime (thermal timescale), we get:

$$\frac{d \log Q_+}{d \log T} = 1 + 7\mu \frac{1 - \beta}{1 + \beta} \quad (18)$$

where $\beta = \frac{P_{gas}}{P_{tot}}$. Finally, from Eq. (9) and Eq. (15), we have:

$$\frac{d \log Q_+}{d \log T} > 4, \quad (19)$$

which is fullfilled if the condition:

$$\beta < \frac{7\mu - 3}{7\mu + 3} \quad (20)$$

is satisfied [4]. This gives the necessary condition for the instability for the case of μ-model, so that the instability occurs only if $\mu > 3/7$.

Magnetized disks:

Strong magnetic field can stabilize the disk [9] We assume magnetic contribution to the pressure

$$P = P_{\text{rad}} + P_{\text{gas}} + P_{\text{mag}}. \quad (21)$$

Let us define the disk magnetization coefficient $\beta' = \frac{P_{mag}}{P_{tot}}$. We put the formula (21) into the Shakura-Sunyaev stress-energy tensor (i.e., for $\mu = 0$ in Eq. (11)), and then we get:

$$\frac{d \log Q_+}{d \log T} = 8(1 - \beta'). \quad (22)$$

$\beta' > \frac{1}{2}$ - disk is stable$\frac{d \log Q_+}{d \log T} \leq \frac{d \log Q_-}{d \log T}$. From the formula (20) we can connect β' and μ as follows:

$$\mu = 1 - \frac{8}{7}\beta'. \quad (23)$$

β is equal to the magnetization parameter - the μ prescription may be connected with magnetization.

Model grid results

Mass, amplitude and period

Result from the model grids:

$$\log P[s] \approx 0.83 \log A + 1.15 \log M + 0.40 \quad (24)$$

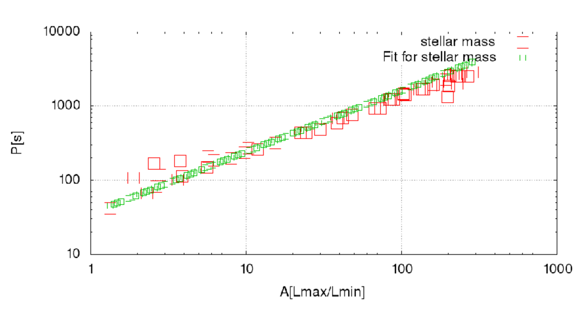


Figure : Amplitude-period correlation for $M = 10M_\odot$ (microquasar mass)

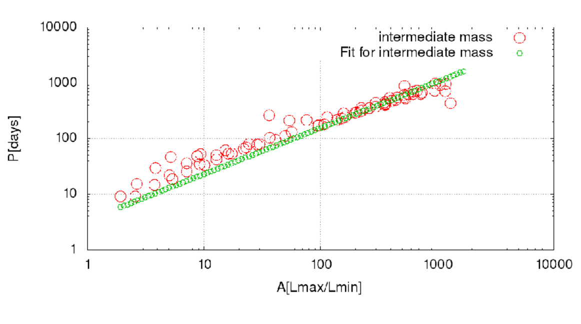


Figure : Amplitude-period correlation for $M = 3 \times 10^4 M_\odot$ (IMBH mass)

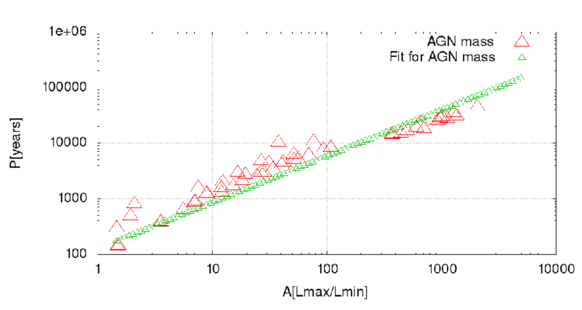
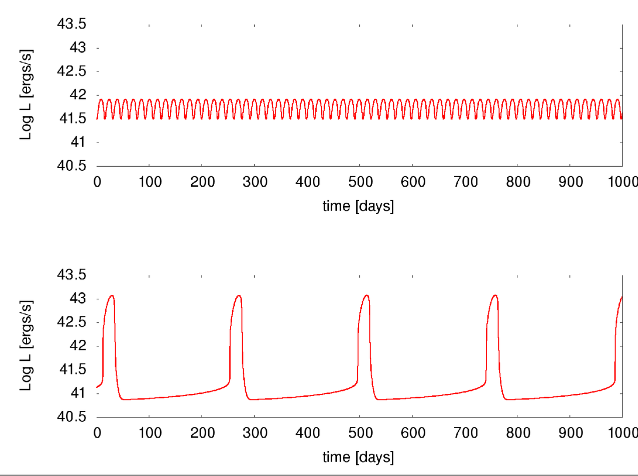


Figure : Amplitude-period correlation for $M = 10^8 M_\odot$ (AGN mass)

Nonlinearity in the lightcurves

Left: comparison between ligthcurves for different values of μ Right : comparison between duty cycles



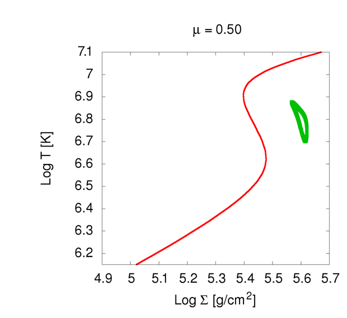


Figure : Flickering duty cycle

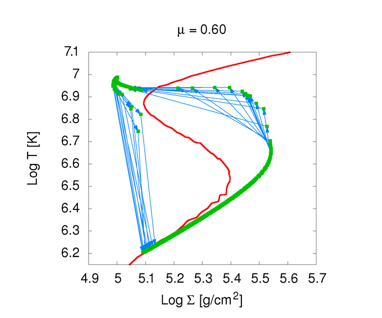
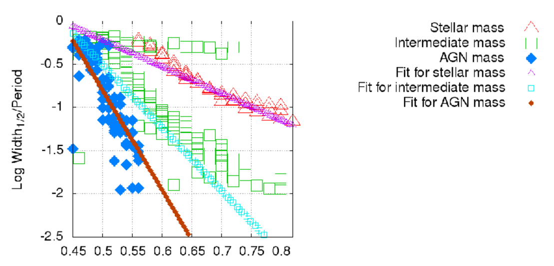


Figure : Burst duty cycle

Lightcurve width

We have strong correlation between μ parameter and shape:

$$-\log \Delta = \frac{1.9 + 1.2 \log M}{\mu - 3/7} \quad (25)$$



Amplitude maps

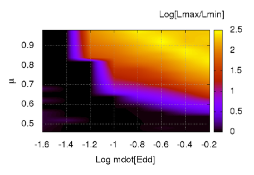


Figure : amplitude map for $M = 10M_\odot$

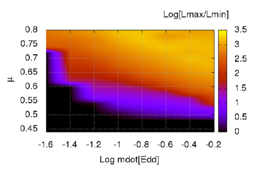


Figure : amplitude map for $M = 3 \times 10^4 M_\odot$

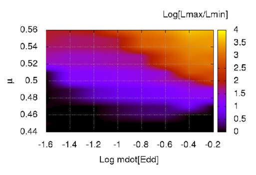


Figure : amplitude map for $M = 10^8 M_\odot$

Accretion rate-amplitude

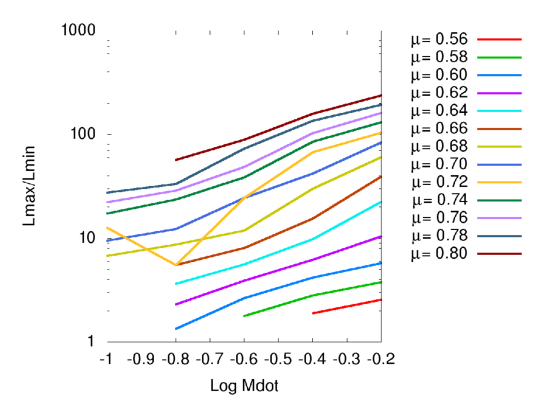


Figure : $M = 10M_\odot$

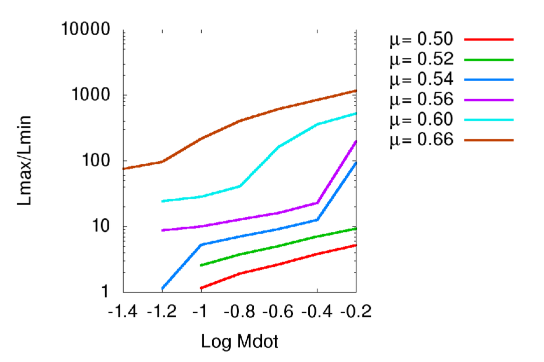


Figure : $M = 3 \times 10^4 M_\odot$

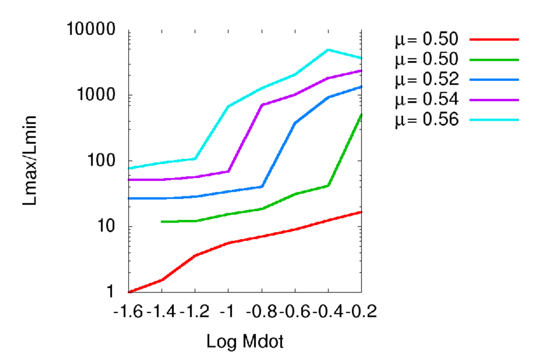
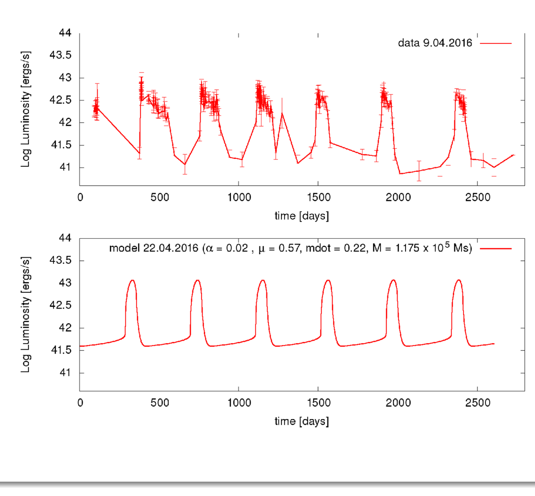


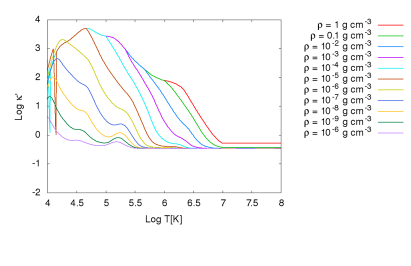
Figure : $M = 10^8 M_\odot$

HLX-1 - mass determination

- Ultraluminous X-ray source in ESO 243-49
- outbursts once per 400 days
- Intermediate-mass black hole candidate
- best model parameters: $M = 1.175 \times 10^5 M_\odot$, $\mu = 0.57$, $\dot{m} = 0.22$



Atomic opacities and thermal instability



- Negative derivative stabilize the disk

$$\frac{d \log Q_-}{d \log T} = 4 - \frac{d \log \kappa}{d \log T} \quad (26)$$

- *The universal "heartbeat" oscillations in black hole systems accross the mass-scale* Wu et. al (2016) https://arxiv.org/abs/1610.04402
- *Modified viscosity in accretion disks. Application to Galactic black hole binaries, intermediate mass black holes and AGN* Grzedzielski et. al (2016) https://arxiv.org/abs/1609.09322

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