Warszawa, 05.07.2018

Topology of configuration spaces for particles on graphs

Tomasz Maciążek
supervisor: dr hab. Adam Sawicki, prof. CTP PAS

Abstract

We study non-abelian quantum statistics on graphs via certain topological invariants, which are the homology groups of configuration spaces. In the first part of this thesis, we formulate a general framework for studying quantum statistics of particles constrained to move in a topological space X. The framework involves the study of flat complex vector bundles over the space of unordered tuples of points from X, known as the configuration space of X. In the second part, we apply this methodology for configuration spaces of graphs. In particular, we use discrete models of graph configuration spaces, which are due to Świątkowski and Abrams. The discrete models are CW complexes, who carry all information about the topology of graph configuration spaces. This allows us to use the tools from algebraic topology to compute the homology groups of graph configuration spaces for some families of graphs. These families are i) tree graphs, ii) wheel graphs, iii) complete bipartite graphs $K_{2,p}$ and $K_{3,3}$. We also describe the generators of the second homology group of simple graphs. Moreover, we compute the homology groups of graph configuration spaces for some small canonical graphs via the discrete Morse theory. As a conclusion, we provide families of graphs, which are good candidates for simplified models in the further study of quantum statistical phenomena and as such may find use for example in anyonic quantum computations.

Tomasz Maciążek