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REFEREE REPORT ON THE PHD THESIS OF TOMASZ MACIĄŻEK
"TOPOLOGY OF CONFIGURATION SPACES FOR PARTICLES ON GRAPHS"

The PhD thesis "Topology of configuration spaces for particles on graphs" has been written by Tomasz Maciążek under the supervision of dr hab. Adam Sawicki, and submitted for review at the Center for Theoretical Physics of the Polish Academy of Sciences in June 2018.

As the title indicates, the thesis is concerned with topology of configuration spaces of particles on graphs – and more precisely with (co)homology groups of such spaces, whose properties are related to quantum statistics of particles on graphs, which on the other hand provides physical motivation to study such systems. The thesis is written in English, it has 5+115 pages, and consists of 6 chapters (which include Introduction and Summary) and Bibliography. Chapters 2, 3, and 4 largely consist of a mathematical introduction to CW-complexes, vector bundles, and configuration spaces of graphs, and introduce tools of which the author takes advantage in chapter 5, where new computational results are presented. The Bibliography consists of 80 references, which include classical publications and textbooks devoted to various general mathematical structures (e.g. by Atiyah and Hirzebruch, Karoubi, Milnor and Stasheff, Whitehead, Spanier, Vainshtein, Hatcher, Kuratowski, Kobayashi and Nomizu, Steenrod, Spivak), classical papers on quantum properties and statistics of particles (e.g. by Leinaas and Myrheim, Laidlaw and DeWitt, Tolar, Wilczek, Kitaev), publications devoted to properties of graphs (e.g. by Świątkowski and Abrams), and quite recent papers about statistics on graphs (e.g. by Harrison, Keating, Robbins, and Sawicki). I find the choice of references in the Bibliography relevant and complete. Overall, the structure of the thesis is clear and logical, and it appears to be written with taking care of the details.

Some results of the thesis are presented in an article by the author and his supervisor published in J. Math. Phys. (2017), and they are also reviewed in conference proceedings published in Acta Phys. Polon. (2017) by the same two authors. Notably, Tomasz Maciążek is also an author of 7 other papers in mathematical physics, out of which 3 have not been coauthored by his supervisor.

The main achievement of the thesis is developing the framework for the analysis of quantum statistics of particles moving in a space of non-trivial topology, using this framework in the context of particles on graphs, and in particular computation of homology groups of graph configuration spaces for various families of graphs. As the author argues, such homology groups indeed encode certain information about quantum statistics, which provides the main physical motivation for their analysis. The author conducts more explicit and detailed analysis for graphs, as in this case various computations are much more tractable than for other topological spaces. Nonetheless, despite simplifications that occur for graphs, the analysis presented by the author still relies on highly sophisticated mathematical tools. Such mathematical background is introduced in chapters 2, 3 and 4 of the thesis. In particular, in chapters 2 and 3 more standard and general topics are summarized, such as homotopy and homology and cohomology theory of CW-complexes, homological exact sequences, structure of vector bundles, Chern classes, and elements of K-theory. Chapter 4 is devoted to a more specialized topic of configuration spaces of graphs, and in particular it summarizes properties of discrete models of graph configuration spaces formulated by Abram and Świątkowski. Such discrete models take form of CW-complexes that are deformation retracts of graph configuration spaces. Analysis of such discrete models by means of standard techniques in algebraic topology enables then the computation of homology groups of graph configuration spaces. Such homology groups are determined in chapter 5, which presents new computational results obtained by the author. The computations presented in this chapter are either analytical, or numerical and based on a computer code developed by the author. The author computes Betti numbers and characterizes homology groups for so called tree graphs, wheel graphs, a family of K(2,p) graphs, and K(3,3) graph. The results of these computations also support the regularity conjecture, which states that Betti numbers of graph configuration spaces grow polynomially with the number of particles.

The results presented in the thesis are very interesting and rely on deep mathematical analysis. Indeed, the author must have got acquainted with a vast scope of topics in homological algebra and algebraic topology – which go much beyond the standard toolkit of a theoretical physicist – and then adjust them to his needs and use to solve specific problems. The author should be complimented on this achievement. Deep knowledge of the above mentioned fields of mathematics should also provide an excellent basis for research in many other areas in mathematical physics. The subject of the thesis provides also a nice example of research area that is interesting to, and should attract both physicists and mathematicians, and motivate their interactions. In particular, apart from physical motivations, the computation of homology groups and Betti numbers in chapter 5 should be of immediate interest to mathematicians too.

While my impression of the thesis is very positive, I would also like to raise a couple of questions or concerns. First, highly mathematical character of the thesis is on one hand advantageous as summarized above, but on the other hand – and as for the thesis in physics – to some extent it seems to dominate physical discussion or conclusions. For example, physical meaning of non-trivial higher homology group, or possible applications or physical realizations of new types of quantum statistics might deserve more extensive discussion.

Similarly, out of three open problems mentioned in the Summary chapter, only one related to quantum computation is presented in a way that makes direct contact with physical applications, while the other ones sound purely mathematically.

The author might also comment on the choice of the topic and material presented in the thesis, in particular in view of many other papers that he wrote and which – as it is claimed – are not immediately related to the contents of this thesis. Do some results in those other papers take advantage of some tools, or rely on some techniques developed in this thesis? On the other hand, out of the results presented in the thesis, only those in section 5.3 seem to be presented in the form of a publication. Are other results found in the thesis supposed to be published in some form too?

To sum up, Tomasz Maciążek has presented a very interesting and well written thesis in mathematical physics, whose results rely on deep results in algebraic topology, and apart from physicists working on quantum statistics should be of interest also to mathematicians. My overall grade of the thesis is high, it meets all formal requirements, and I recommend moving on to the next stages of the doctoral process.