

Research project *Uncertainty relations and quantum entanglement*  
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is realized by the Consortium of Faculty of Physics, Astronomy and Applied  
Information Science at the Jagiellonian University in Cracow and  
Center for Theoretical Physics, Polish Academy of Sciences in Warsaw

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Quantum uncertainty relations, originally due to Heisenberg, Kennard and Schrödinger, which provide fundamental bounds for variance of outcomes of two quantum measurements, belong to the cornerstones of the quantum theory. During recent years several new generalizations and improvements of these classical results were obtained. In particular, uncertainty relations were formulated in terms of the entropies, which characterize outcomes of quantum measurements.

Quantum mechanics is a linear theory and it admits existence of superposition of quantum states. In the case of quantum systems composed of several subsystems there exists so called entangled states, which display non-classical correlations between the subsystems. Such peculiar quantum states, already constructed in laboratories, are crucial for realization of quantum cryptography, quantum teleportation and several other tasks of quantum information processing.

Main goal of the project is to derive novel entropic uncertainty relations concerning two measurements of quantum systems described by pure states in a finite dimensional Hilbert space and to generalize obtained results for the case of arbitrary number of measurements, mixed states and generalized quantum measurements. In particular, we will establish uncertainty relations dedicated to composed quantum systems, for which the role of the effects of quantum entanglement will be analyzed and explained.

We plan to investigate also entropic certainty relations – upper bounds for the average entropy characterizing the outcomes of a quantum measurement and distinguished quantum states, for which these relations are saturated. Results obtained will be applied to design schemes of optimal quantum measurements and to analyze theoretical principles of quantum memories. A parallel aim of the project is to analyze entanglement in multipartite quantum systems, to identify cases for which there exist states maximally entangled with respect to every partition of the system into two parts.

The first phase of the project we will start with numerical simulations, which will allow us to identify, which conjectured improvements of the known certainty / uncertainty relations might be true. In the subsequent phase of the

project we will attempt to prove generalized uncertainty relations using algebraic methods and techniques of functional analysis and operator theory. In parallel we shall pursue the complementary goal of the project aiming to construct maximally entangled quantum states in multipartite setup and to analyze their properties using combinatorial, algebraic and geometric methods. Results obtained will allow us for a better understanding of the phenomenon of quantum entanglement in multipartite systems, for constructions of optimal schemes of quantum measurements and for other applications in the theory of quantum information processing.