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Universality in quantum computation

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Abstract

This thesis is devoted to one of the most important problems in quantum computing, i.e. the universality problem. It reduces to deciding, if a finite set of gates operating on a system of qudits allows to construct an arbitrary operation on the qudits with an arbitrarily small error. This problem has a great practical significance as only a few quantum gates can be realized experimentally quite easily. From a mathematical point of view, universality is a problem of generating an infinite group, like a group of unitary or orthogonal transformations, by a finite number of group elements.

This thesis is a part of a project financed by the National Science Center and devoted to universality, optimality and control in quantum computing DEC-2015/18/E/ST1/00200. The main aim of our study was to formulate an algorithm for deciding universality of an arbitrary set of quantum gates. We required from our algorithm to be easy to apply and return the answer after a finite number of iterations.

Our thesis is organized as follows. In Chapter 1 we present basic concepts from theory of quantum computing. We describe a simple model of a quantum computer and give a mathematically strict definition of universality. In this chapter we also present a detailed structure of this thesis. Chapter 2 includes all the mathematical concepts that are used in this thesis. In Chapters 3 and 4, that are the core of this thesis, we present the results of our study. Chapter 3 includes a method that allows to decide universality for particular sets of one-qubit gates, whereas the approach presented in Chapter 4 can be applied for arbitrary set of quantum gates. The mathematical tools used in both chapters comes from various areas of mathematics, from field theory to representation theory of compact, semisimple Lie groups. Finally, we include in Appendix some results that are outside main thread of this thesis, but which supplement the main results.

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